嘉南藥理科技大學
Chia Nan University of Pharmacy & Science

103 年度校內教師專題研究計畫之研究報告

論文題目**:**

伴隨加拿大股票市場門檻之三股票市場報酬波動的動態關聯性分析: 台灣、韓國及新加坡股票市場之實證研究

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計畫編號**: CN10320**

中華民國 **103** 年 **12** 月 **31** 日

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Dynamic Relatedness Analysis of Three Stock Market Return Volatility with a Threshold of Canada Stock Market: Evidence of Taiwan, Korea, and Singapore Countries

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Abstract

This paper studies the associations among and the model construction of Taiwan, Korea, and Singapore’s stock markets during the period from January 2003 to December 2013. In this paper we construct a dynamic conditional correlation (DCC) and a trivariate AGARCH (1, 1) model to evaluate the associations, and find that there does exist an asymmetrical effect among the three stock markets with a factor of Canada stock market. The result of empirical correlation analyses also shows that Taiwan’s stock market returns positively affect the Korea and Singapore stock market returns, and the volatility of the three stock market returns interact with one another. Furthermore, the time lags of Taiwan stock market returns do not affect the returns of the Korea and Singapore stock markets. The variation risk of the Canada’s stock market returns’ volatility affects the variation risks of Taiwan, Korea and Singapore stock market returns. Empirical results also show that both good and bad news of the Canada stock market will actually affect the variation risks of those three stock market returns. Therefore, based on the viewpoint of DCC, the explanatory ability of the trivariate AIGARCH(1, 1) model is better than the traditional model of the trivariate GARCH. The evidence suggests that stock market investors or international fund managers must evaluate the variation risk and relationships of the stock market returns’ volatility.

Keywords: Stock market returns, trivariate GARCH model, asymmetrical effect, trivariate asymmetric GARCH model, DCC.

1. Introduction

With greater internationalization and liberalization, international investment and the worldwide circulation of capital have been increasing, resulting in close relationships among countries and their respective stock markets. We know that Taiwan already established sound economic regulations when she was under the control of the U.K., and currently Taiwan is an important economic and trade area for mainland China. According to the statistics of the Securities and Futures Commission (SFC), at the end of December 2005, the Hong Kong exchange market's turnover was up to US$1,055 billion, the eighth highest in the world and the second highest in Asia. Taiwan undoubtedly also plays a very important role in the global economic and financial system. We also know that the economic growth rate of Singapore, one of the Four Asian Tigers, was 7.9% in 2006, and is forecast to expand at a rate of 5.5-7.5% in the future. Moreover, Singapore is the main financial center of Asia, and her foreign exchange market is the fourth largest trading market in the world. In addition, Taiwan and Singapore have a close relationship with Korea, and the three stock markets are the most important financial markets in Asia. How these three stock markets impact one another certainly merits further discussion. Besides, we also further considers an influence factor of Canada stock market in this paper.

In the financial time series non-linear research literature, Engle (1982) proposed the autoregressive conditionally heteroskedasticity (ARCH) model, and Bollerslev (1986) presented the generalization autoregressive conditionally heteroskedasticity (GARCH) model. These two models can find financial properties when the conditional variance is not a fixed parameter. Nelson (1990) considered fluctuations in stock prices and found that they have both positive and negative correlations with future stock price volatility. The GARCH model supposes a settled time conditional variance for the preceding issue of conditional variance and an error term square function. Therefore, the error term's positive and negative values do not respond to its influence on the conditional variance equation. The conditional variance only changes when the error term's value changes, and cannot go along with the error term's positive and negative changes. To correct this flaw, Schwert (1990) and Nelson (1991) presented an exponential GARCH model, and Glosten, Jaganathan, and Runkle (1993) developed a GJR-GARCH model. These are termed asymmetric GARCH models. There have been many research studies on the asymmetric problems, such as Horng and Lee (2008), Brooks (2001), Poon and Fung (2000), Christie (1982), French, Schwert, and Stambaugh (1987), Campell and Hentschel (1992), Koutmos and Booth (1995), and Koutmos (1996). These studies expanded the research methods used in the area of return volatility of stock markets. For statements on the multivariate GARCH model was proposed, scholars such as Yang and Doong (2005), Yang (2004), Granger, Hung and Yang (2002) and Bollerslev (1990) developed the bivariate GARCH model.

The main goal of this paper is to discuss the associations between Taiwan, Korea, and Singapore's stock return volatility. The paper constructs a trivariate GARCH theoretical model and examines whether or not there is an asymmetrical influence among the three stock markets. And we will also further discuss the influence of the Canada stock market on the three stock markets. The organization of this paper is as follows. Section 2 describes the data characteristics of the three stock markets. Section 3 introduces the models of GARCH and trivatiate GARCH. Section 4 presents the empirical results of the trivatiate GARCH model. Section 5 presents the asymmetrical test of the trivatiate GARCH model. Section 6 presents the asymmetrical and the trivatiate GARCH model, and the last section presents conclusions and recommendations.

2. Data Characteristics

2.1 Data Sources

This study uses the Taiwan weighted stock index, the Singapore Straits Times index, the Korea KOSPI index and Toronto 300 stock index as the sample. The data period is from January 2003 to December 2013, and uses the date data for all stock price indices, with the data are obtained from the Taiwan Economic Journal (TEJ), a database in Taiwan. For treat the data processing, we have already considered the stock markets' common trading days, so the sample size is 2511 for all three stock markets, respectively.

2.2 Returns Calculation and Trend of Charts

To compute the stock price index return rates, this paper adopts the natural logarithm of the stock price index for every stock market sample (TW_t, $SING_t$, $KOREA_t$, $CANA_t$) with one step difference and then multiplied by 100, i.e. for the Taiwan stock market, the stock price index return rates are $RTW_t = 100*[\ln(TW_t / TW_{t-1})]$. For the Singapore stock market, the stock price index return rates are $RSING_t = 100*[\ln(SING_t / SING_{t-1})]$. For the Korea stock market, the stock price index return rates are $RKOREA_t = 100*[\ln(KOREA_t / KOREA_{t-1})]$. For the Canada stock market, the stock price index return rates are $RCANA_t = 100*[\ln(CANA_t / CANA_{t-1})]$. Figure 1 shows the trend charts of Taiwan, Korea, Singapore and Canada tock price indices during the sample period. Figure 2 shows those of the Taiwan, Korea, Singapore and Canada stock price index return rates.

From Figure 1, we can see that Taiwan, Korea, Singapore and Canada stock price indices clearly display the same trends. From Figure 2, we also know that the clustering phenomenon is present in the volatility of these three stock market returns. We may also know that Taiwan, Korea and Singapore's stock markets have a certain relationship in their return volatility processes. And Canada stock market returns may affect the return volatility of these three stock market returns. This also shows that there are mutual relationships in stock returns among those three markets. This is one of main motivations for discussing the relationships between Taiwan, Korea and Singapore's stock price returns with a factor of Canada stock market.

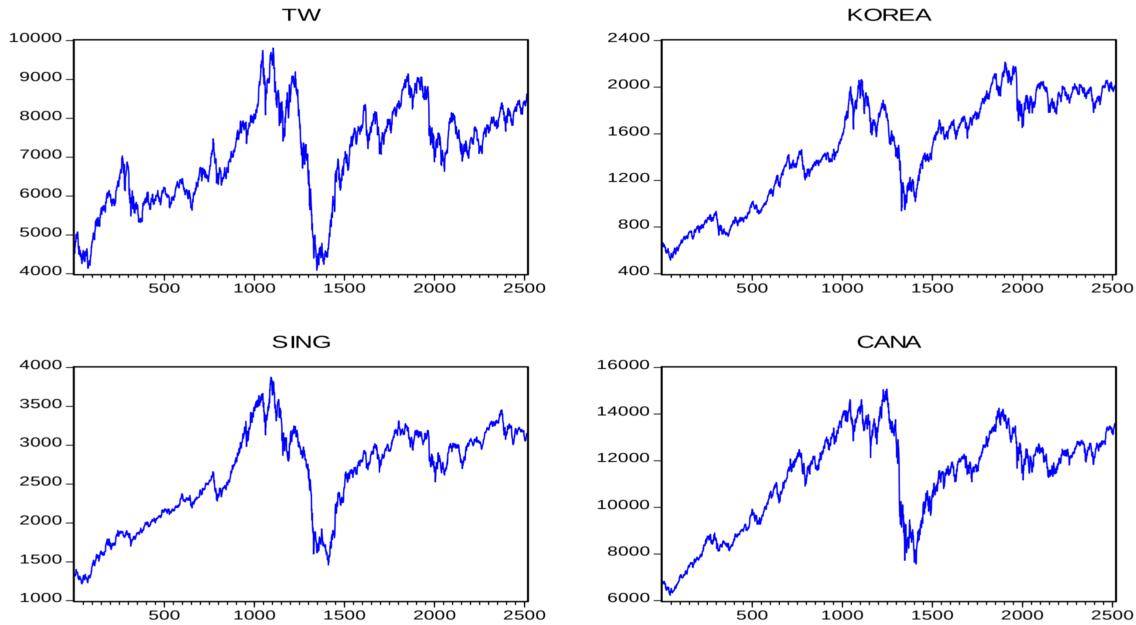

Figure 1.Trend charts of Taiwan, Korea, Singapore and Canada stock price indices

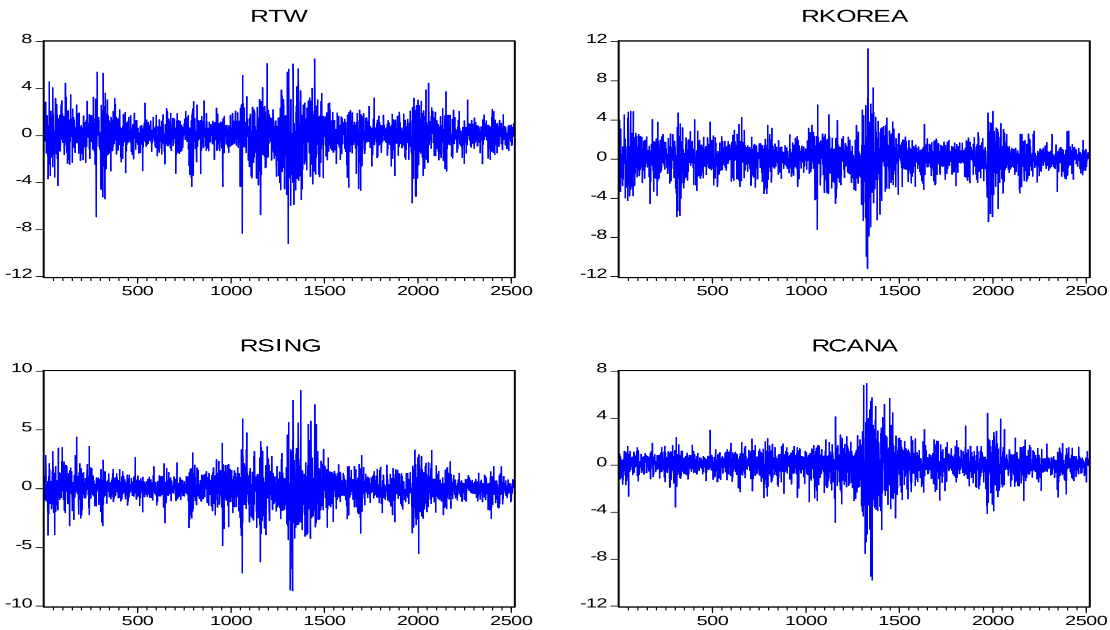

Figure 2.Trend charts of return rates of Taiwan, Korea, Singapore and Canada stock price indices

2.3 Statistics

Table 1-1 and 1-2 shows some basic statistical analyses: mean value, standard deviation, kurtosis coefficient, skewed coefficient, and normal distribution examination. As shown in Table 1-2, the average return rate of the Taiwan stock price index is 0.0257, the average return rate of the Singapore stock price index is 0.0342, and the average return rate of the

Korea stock price index is 0.0459. The variation risk of the Taiwan stock price index return rate is 1.3874, the variation risk of the Singapore stock price index return rate is 1.2433, and the variation risk of the Korea stock price index return rate is 1.5069, making the variation risk of the Korea's stock price index return rate the highest. From the normal distribution test of Jarque-Bera, we know that the three stock price return rates are not a normal distribution. Note that the kurtosis coefficients of three sequences are larger than 3, and this demonstrates that the study data reflect the heavy tail distribution phenomenon. We know that as the sample is large enough, the analytical result of the heavy tail distribution approaches that of the normal distribution. The stock price return rates for the Taiwan, Korea, Singapore and Canada markets show a stationary state sequence, as shown below in Table 2.

Table 1-1. Statistical Data

Statistic	TW	KOREA	SING	CANA
Mean	7059.711	1474.163	2601.804	11251.43
S-D	1251.535	457.3603	611.1794	2109.180
Skewed	-0.331088	-0.405946	-0.431328	-0.614135
Kurtosis	2.391989	1.873135	2.169336	2.358257
J-B N	84.5532***	201.8208***	150.0510***	200.9304***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
sample	2511	2511	2511	2511

Notes: (1) S-D denotes the standard deviation of data.

(2) J-B N denotes the Jarque-Bera normal distribution test.

(3) p-value < α denotes significance (α =1%, α =5%, α =10%).

*** denotes significance at the level 1%.

Table 1-2. Statistical Data

Statistic	RTW	RKOREA	RSING	RCANA
Mean	0.025693	0.045923	0.034214	0.027914
S-D	1.387351	1.506871	1.243252	1.169550
Skewed	-0.486699	-0.461242	-0.186466	-0.733388
Kurtosis	6.957505	8.502771	9.674562	11.36668
J-B N	1737.061***	3255.832***	4673.710***	7545.968***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
sample	2510	2510	2510	2510

Notes: (1) S-D denotes the standard deviation of data.

(2) J-B N denotes the Jarque-Bera normal distribution test.

(3) p-value < α denotes significance (α =1%, α =5%, α =10%).

*** denotes significance at the level 1%.

2.4 Unit Root Test

In order to find a suitable model, one may first determine the stability of the time series data, as well as avoid the non-stationary state of the time series sequences and reduce the mistake of the empirical result. To do so, this paper further uses the unit root test of Dickey-Fuller (1979 and 1981, ADF) and KSS (Kapetanios et al., 2003) to determine the stability of the time series data. The ADF and KSS examination results are listed in Table 2, and show that the Taiwan, Korea, Singapore and Canada's stock price indices have the unit root characteristic. This indicates that the time series data is not stable. Using a step difference of the time series data, under α =1% significance level, the stock returns of the Taiwan, Korea, Singapore and Canada's stock markets have a stationary sequence. Therefore, we can further analyze the time series data of the three stock markets.

Table 2. ADF and KSS-Unit root test of the data

ADF	RTW	RKOREA	RSING	RCANA
Statistic	-12.7462***	-30.1898***	-48.5474***	-9.5263***
C-V	-3.9617	-3.4116	-3.1277	
(S-L)	(α =1%)	(α =5%)	(α =10%)	
KSS	**RTW**	**RKOREA**	**RSING**	**RCANA**
Statistic	-23.3081***	-21.4943***	-24.2396***	-23.7219***
C-V	-2.82	-2.22	-1.92	
(S-L)	(α =1%)	(α =5%)	(α =10%)	

Notes：(1) C-V denotes the critical value and S-L denotes significance level.

(2) *** denotes significance at the level 1%.

2.5 Co-integration Test

From the co-integration test of Johansen (1991), we know that the statistics of λ_{max} and the Trace are not significant under the level of 5% in Table 3. This demonstrates that these four stock markets do not have co-integrated relations. In Table 4, the unconditional correlation matrix of Taiwan, Korea, Singapore and Canada stock market returns shows correlation. Although the Taiwan, Korea, Singapore and Canada stock markets do not have long-term co-integrated relations, those four markets actually do affect one another. Therefore, we investigate further to understand the relations between the three stock markets with a factor of Canada stock market.

Table 3. Johansen's Co-integration test (the lag of VAR is 6)

Null (H_0)	λ_{max}	C-V ($\alpha = 5\%$)	Trace	C-V ($\alpha = 5\%$)
None	23.6042	32.1183	62.8759	63.8761

At most 1	19.5929	25.8232	39.2718	42.9153
At most 2	13.1115	19.3870	19.6789	25.8721
At most 3	6.5674	12.5180	6.5674	12.5180

Notes：(1) C-V denotes the critical value.

(2) The lag of VAR is selected by the BIC rule (Schwartz, 1978).

Table 4. Unconditional Correlation Matrix

coefficient	TW	KOREA	SING	CANA
Hk	1	0.8699	0.9383	0.8987
KOREA	0.8699	1	0.9138	0.8878
SING	0.9383	0.9138	1	0.9503
CANA	0.8987	0.8878	0.9503	1
coefficient	RTW	RKOREA	RSING	RCANA
RHK	1	0.7000	0.6156	0.2548
RKOREA	0.7000	1	0.6515	0.2880
RSING	0.6156	0.6515	1	0.3599
RCANA	0.2548	0.2880	0.3599	1

2.6 ARCH Effect Test

The ARCH effect test is used to determine stock return volatility and whether the conditional heteroskedasticity phenomenon is present. We also use the Ljung-Box (1978) test method, the Lagrange Multiplier (LM) test method of Engle (1982), and the F distribution test method of Tsay (2004) to further confirm the variance of the residual error sequence and whether or not there is an ARCH effect. If there is an ARCH effect, we use the GARCH model to match it suitably. The ARCH effect test takes the residual error square of the past q periods to carry out regression analysis. The ARCH effect test is based on the AR model in Table 8. Its mathematical form is:

$$\hat{a}_t^2 = d_0 + d_1\hat{a}_{t-1}^2 + \cdots + d_q\hat{a}_{t-q}^2 + v_t \tag{1}$$

We test the null hypotheses $H_0 : d_1 = d_2 = \cdots = d_q = 0$ by (1). When H_0 is rejected, it means that the ARCH effect does exist, and so we can use the GARCH model to fit it.

We implement the LM, F, and Ljung-Box (L-B) test methods to examine the stock price date returns and to determine whether or not there is a conditional heteroskedasticity phenomenon. The results of the ARCH effect test for the three stock markets are listed in Table 5-7. The results show that the analytical model of the three stock price return rates has a significant statistical value under the level α =5% and a conditional heteroskedasticity phenomenon exists. This suggests that it is a suitable match, and the GARCH model could be used to analyze data.

Table 5. ARCH Effect Test for Taiwan (lag=30)

Engle LM test		Tsay F test		L-B test	
				$LB^2(2)$	$LB^2(3)$
Statistic	298.6583***	Statistic	11.1774***	4.0124***	4.0801***
(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0001)	(0.0000)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

Table 6. ARCH Effect Test for Korea (lag=30)

Engle LM test		Tsay F test		L-B test	
				$LB^2(1)$	$LB^2(2)$
Statistic	609.4573***	Statistic	26.6010***	7.0761***	3.8073***
(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0000)	(0.0001)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

Table 7. ARCH Effect Test for Singapore (lag=30)

Engle LM test		Tsay F test		L-B test	
				$LB^2(2)$	$LB^2(3)$
Statistic	534.4147***	Statistic	22.4255***	6.6760***	6.1504***
(p-value)	(0.0000)	(p-value)	(0.0000)	(0.0000)	(0.0000)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

3. GARCH and Trivariate GARCH Models

In this section, we introduce models for the GARCH and the trivariate GARCH. If we use only the single variable GARCH model to analyze the relatedness of Taiwan, Korea and Singapore's stock price return volatility, then we allow stock return volatility to change only with time. We disregard the variance structure of these markets' stock price return volatility. The analysis also produces inefficiency and bias in the model estimation. In fact, the conditional variances of the three stock return volatilities are all favored along with the time change, and the trivariate GARCH model can simultaneously consider the time dependence of the volatility of the three stock markets. Therefore, we use the trivariate GARCH model to discuss the three stock price markets' relations and their impact on the returns of the Taiwan, Korea and Singapore's stock markets. The models for the GARCH and the trivariate GARCH are outlined below. .

3.1 GARCH Model

Based on the ARCH model (Engle, 1982), Bollerslev (1986) proposes the generalized autoregressive conditional heteroskedasticity (GARCH) model. The GARCH model allows conditional variance to depend on the residual error square and the function of past conditional variance, and makes the conditional variance into a dynamic structural model. This model thus achieves the two goals of flexibility and simplification. The general form of GARCH (p, q) is as follows:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{2}$$

$$a_t \mid \Omega_{t-1} \sim N(0, h_t), \quad \alpha_i \geq 0, \beta_j \geq 0, \alpha_0 > 0$$

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \text{ ， } i = 1,2,\cdots,q \text{ and } j = 1,2,\cdots,p$$

where Ω_{t-1} is from all past information from period 1 to period t-1 in the set; h_t is the conditional variance that depends on the residual error square of the past q periods and its conditional variance of p periods; ($\alpha_0, \alpha_1, \cdots, \alpha_q$) and ($\beta_1, \cdots, \beta_p$) are the unknown parameter vectors; a_t is a disturbance item (or white noise), $N(0, h_t)$ denotes the normal distribution with mean equal to 0 and variance equal to h_t.

The main difference between the GARCH model and the ARCH model is the conditional variance. In addition to the influence of the residual error square, the GARCH model also is affected by the conditional variance with the influence of lag periods. Therefore, the GARCH model has a general special characteristic, and the ARCH model does not. The structure of the conditional variance not only has flexibility, also has more applications. The ARCH model is the only special case of the GARCH model- namely, when p=0, the GARCH(p, q) model turns into the ARCH(q) model.

We know that the GARCH (p, q) model is equal to the ARCH(∞) model and the parameters to be estimated can be reduced. However, unlike the ARCH model, the GARCH model requires that the estimated coefficient be positive. Moreover, in many finance time series, data volatility shows asymmetrical characteristics. To solve this problem, for example, Glosten, Jagannathan and Runkle (1993) developed a GJR-GARCH model, the details of which will not be elaborated on in this paper. Compared with the GARCH model, the merits of the GJR-GARCH model are in distinguishing the different influences of good news and bad news.

3.2 Trivariate GARCH Model

From Table 1 to Table 7, as noted above, we also know that Taiwan, Singapore, and Korea's stock price returns have heteroskedasticity, leptokurtic distribution, and stationary sequence statistical characteristics. Therefore, we use the trivariate GARCH model to analyze the influence of returns in these stock markets. The trivariate GARCH model refers to that in Tsay (2004). This model will be used to discuss the relationships between the volatility of Taiwan, Korea and Singapore’s stock price returns. The construction of the GARCH(1, 1) model is as follows:

$$RTW_t = \phi_0 + \sum_{j=1}^{n} \phi_{1j} RTW_{t-j} + \sum_{j=1}^{n} \phi_{2j} RKOREA_{t-j} + \sum_{j=1}^{n} \phi_{3j} RSING_{t-j} + \sum_{j=1}^{n} \phi_{4j} RCANA_{t-j} + a_{1,t} \tag{3}$$

$$RKOREA_t = \varphi_0 + \sum_{j=1}^{n} \varphi_{1j} RTW_{t-j} + \sum_{j=1}^{n} \varphi_{2j} RKOREA_{t-j} + \sum_{j=1}^{n} \varphi_{3j} RSING_{t-j} {}_{2,t} + \sum_{j=1}^{n} \varphi_{4j} RCANA_{t-j} + a_{2,t} \tag{4}$$

$$RSING_t = \psi_0 + \sum_{j=1}^{n} \psi_{1j} RTW_{t-j} + \sum_{j=1}^{n} \psi_{2j} RKOREA_{t-j} + \sum_{j=1}^{n} \psi_{3j} RSING_{t-j} + \sum_{j=1}^{n} \psi_{4j} RCANA_{t-j} + a_{3,t} \tag{5}$$

$$h_{11,t} = \alpha_{10} + \alpha_{11} a_{1,t-1}^2 + \beta_{11} h_{11,t-1} + \eta_1 RCANA_{t-1}^2 \tag{6}$$

$$h_{22,t} = \alpha_{20} + \alpha_{21} a_{2,t-1}^2 + \beta_{21} h_{22,t-1} + \eta_2 RCANA_{t-1}^2 \tag{7}$$

$$h_{33,t} = \alpha_{30} + \alpha_{31} a_{3,t-1}^2 + \beta_{31} h_{33,t-1} + \eta_3 RCANA_{t-1}^2 \tag{8}$$

$$q_{12,t} = c_0 + c_1 \rho_{12,t-1} + c_2 a_{1,t-1} a_{2,t-1} / \sqrt{h_{11,t-1} h_{22,t-1}} \text{，} \tag{9}$$

$$q_{13,t} = d_0 + d_1 \rho_{13,t-1} + d_2 a_{1,t-1} a_{3,t-1} / \sqrt{h_{11,t-1} h_{33,t-1}} \text{，} \tag{10}$$

$$q_{23,t} = e_0 + e_1 \rho_{23,t-1} + e_2 a_{2,t-1} a_{3,t-1} / \sqrt{h_{22,t-1} h_{33,t-1}} \text{，} \tag{11}$$

$$\rho_{12,t} = \exp(q_{12,t}) / (\exp(q_{12,t}) + 1) \text{，} \tag{12}$$

$$\rho_{13,t} = \exp(q_{13,t}) / (\exp(q_{13,t}) + 1) \text{，} \tag{13}$$

$$\rho_{23,t} = \exp(q_{23,t}) / (\exp(q_{23,t}) + 1) \text{，} \tag{14}$$

$$h_{12,t} = \rho_{12,t} \sqrt{h_{11,t}} \sqrt{h_{22,t}} \tag{15}$$

$$h_{13,t} = \rho_{13,t} \sqrt{h_{11,t}} \sqrt{h_{33,t}} \tag{16}$$

$$h_{23,t} = \rho_{23,t} \sqrt{h_{22,t}} \sqrt{h_{33,t}} \tag{17}$$

$\vec{a}_t' = (a_{1,t}, a_{2,t}, a_{3,t})$ obeys the trivariate normal distribution- namely, $N(\vec{0}, H_t)$, among

$\vec{0}' = (0,0,0)$ and

$$H_t = \begin{bmatrix} h_{11,t} & h_{21,t} & h_{31,t} \\ h_{12,t} & h_{22,t} & h_{32,t} \\ h_{13,t} & h_{23,t} & h_{33,t} \end{bmatrix}, \quad h_{12,t} = h_{21,t}, \quad h_{13,t} = h_{31,t}, \quad h_{23,t} = h_{32,t}.$$

The probability density function of $\vec{a}_t$ is

$$f(a_{1,t}, a_{2,t}, a_{3,t} \mid H_t) = \frac{1}{(2\pi)^{3/2} |H_t|^{1/2}} \exp\left\{ \frac{-1}{2} \vec{a}_t' H_t^{-1} \vec{a}_t \right\} \tag{18}$$

Where $\rho_{12,t}$ is the dynamic conditional correlation (DCC) coefficient of $a_{1,t}$ and $a_{2,t}$, $\rho_{13,t}$ is the DCC coefficient of $a_{1,t}$ and $a_{3,t}$, and $\rho_{23,t}$ is the DCC coefficient of $a_{2,t}$ and $a_{3,t}$. In addition, H_t^{-1} is the inverse matrix of H_t. In this paper, we use the normal distribution for the stochastic error term, and also use the maximum likelihood algorithm method of BHHH (Berndt et. al., 1974) to estimate the parameters of the proposed model.

4. Empirical Results of the Trivatiate GARCH Model

4.1 Trivariate GARCH Model and Parameter Estimation

This section uses the trivariate GARCH model- i.e. (3)-(18) types to analyze the relatedness of Taiwan, Korea and Singapore's stock price return volatilities with a factor of Canada stock market. The parameter estimation first considers a general model and is based on the estimated results. We then delete some explanatory variables that are not significant. Finally, we obtain a simplified model for the analysis of the relatedness of Taiwan, Korea and Singapore stock price return volatilities. The empirical results show that Taiwan, Korea and Singapore's stock price return volatility may be constructed on the trivariate IGARCH(1, 1) model. Its estimated results are in Table 8. The proposed model is diven as follows:

$$RTW_t = \phi_0 + \phi_{11} RTW_{t-1} + \phi_{21} RKOREA_{t-1} + \phi_{31} RSING_{t-1} + \phi_{41} RCANA_{t-1} + a_{1,t}$$

$$RKOREA_t = \varphi_0 + \varphi_{11} RTW_{t-1} + \varphi_{21} RKOREA_{t-1} + \varphi_{31} RSING_{t-1} + \varphi_{41} RCANA_{t-1} + a_{2,t}$$

$$RSING_t = \psi_0 + \psi_{11} RTW_{t-1} + \psi_{21} RKOREA_{t-1} + \psi_{31} RSING_{t-1} + \psi_{41} RCANA_{t-1} + a_{3,t}$$

$$h_{11,t} = \alpha_{10} + \alpha_{11} a_{1,t-1}^2 + \beta_{11} h_{11,t-1} + \eta_1 RCANA_{t-3}^2$$

$$h_{22,t} = \alpha_{20} + \alpha_{21} a_{2,t-1}^2 + \beta_{21} h_{22,t-1} + \eta_2 RCANA_{t-3}^2$$

$$h_{33,t} = \alpha_{30} + \alpha_{31} a_{3,t-1}^2 + \beta_{31} h_{33,t-1} + \eta_3 RCANA_{t-3}^2$$

$$q_{12,t} = c_0 + c_1 \rho_{12,t-1} + c_2 a_{1,t-1} a_{2,t-1} / \sqrt{h_{11,t-1} h_{22,t-1}}$$

$$q_{13,t} = d_0 + d_1 \rho_{13,t-1} + d_2 a_{1,t-1} a_{3,t-1} / \sqrt{h_{11,t-1} h_{33,t-1}}$$

$$q_{23,t} = e_0 + e_1\rho_{23,t-1} + e_2 a_{2,t-1} a_{3,t-1} / \sqrt{h_{22,t-1} h_{33,t-1}}$$

$$\rho_{12,t} = \exp(q_{12,t}) / (\exp(q_{12,t}) + 1)$$

$$\rho_{13,t} = \exp(q_{13,t}) / (\exp(q_{13,t}) + 1)$$

$$\rho_{23,t} = \exp(q_{23,t}) / (\exp(q_{23,t}) + 1)$$

$$h_{12,t} = \rho_{12}\sqrt{h_{11,t}}\sqrt{h_{22,t}}$$

$$h_{13,t} = \rho_{13}\sqrt{h_{11,t}}\sqrt{h_{33,t}}$$

$$h_{23,t} = \rho_{23}\sqrt{h_{22,t}}\sqrt{h_{33,t}}$$

Where $\vec{a}_t' = (a_{1,t}, a_{2,t}, a_{3,t})$ obeys the trivariate normal distribution- namely, $N(\vec{0}, H_t)$, among

$\vec{0}' = (0,0,0)$ and

$$H_t = \begin{bmatrix} h_{11,t} & h_{21,t} & h_{31,t} \\ h_{12,t} & h_{22,t} & h_{32,t} \\ h_{13,t} & h_{23,t} & h_{33,t} \end{bmatrix}, \quad h_{12,t} = h_{21,t}, \quad h_{13,t} = h_{31,t}, \quad h_{23,t} = h_{32,t}.$$

The probability density function of $\vec{a}_t$ is

$$f(a_{1,t}, a_{2,t}, a_{3,t} \mid H_t) = \frac{1}{(2\pi)^{3/2}|H_t|^{1/2}} \exp\left\{\frac{-1}{2}\vec{a}_t' H_t^{-1} \vec{a}_t\right\}$$

Where $\rho_{12,t}$ is the dynamic conditional correlation (DCC) coefficient of $a_{1,t}$ and $a_{2,t}$, $\rho_{13,t}$ is the DCC coefficient of $a_{1,t}$ and $a_{3,t}$, and $\rho_{23,t}$ is the DCC coefficient of $a_{2,t}$ and $a_{3,t}$. In addition, H_t^{-1} is the inverse matrix of H_t. In this paper, we use the normal distribution for the stochastic error term, and also use the maximum likelihood algorithm method of BHHH (Berndt et. al., 1974) to estimate the parameters of the proposed model.

With the estimated results of the trivariate IGARCH(1, 1) model in Table 8, we use a P-value to test whether the estimated value of the parameters' coefficient is significant. During the sample period, Taiwan stock price returns is affected by the constant term. The Taiwan stock price return receives the time lags' influence of the Taiwan and the Korea stock price returns with the time lags equals 1. And the Taiwan stock price return receives the time lags' influence of the Canada stock price returns with the time lags equals 1. The Korea stock price return is affected by the constant term. Korea stock price returns does not receive the influence of Taiwan, Korea and Singapore stock price returns with the time lags equals 1. And the Korea stock price return receives the time lags' influence of the Canada stock price returns with the time lags equals 1. Similarly, Singapore stock price returns is also affected by the constant term. The Singapore stock price return receives the affect of the Korea and Canada stock price returns with the time lags equals 1. The Singapore stock price return does not receive the affect of the Singapore stock price returns with the time lags equals 1.

On the other hand, the correlation coefficient value of Taiwan and Korea stock price return volatilities is significant ($\bar{\rho}_{12}$=0.6086). This result means that Korea stock price returns' volatility has a positive influence on Taiwan stock price returns' volatility, and they have precisely synchronized mutual influence. When the variation risk of Korea stock price returns increases, investors' risk in Taiwan stock price market can increase. Likewise, when the variation risk of the Korea stock price return decreases, investors' risk in the Taiwan stock price market can also decrease. Similarly, the correlation coefficient value of Taiwan and the Singapore stock price return volatilities is significant ($\bar{\rho}_{13}$=0.6868), and the two stock price markets have a high degree of relatedness. This result also shows that Taiwan stock price returns' volatility has a positive influence over Singapore stock price returns' volatility. The correlation coefficient value of the Singapore and the Korea stock price return volatilities is also significant ($\bar{\rho}_{23}$=0.5835). This result also shows that Korea stock price returns' volatility has positive influence on Singapore stock price returns' volatility, and the three stock price markets also have a high degree of relatedness.

The observed conditional variance equation of the estimated coefficient, under the 1% significance level, demonstrates that all the conditional variance estimated coefficients in Table 8 are significant. The empirical results show that the previous one periods' residual error square item and the previous period's conditional variance will affect Taiwan, Korea and Singapore stock price return rate volatilities and can also produce different variation risks. In Table 8, we have the variation risk for the Singapore's stock price market is the lowest ($\beta_{31} = 0.7842$). These three stock markets have fixed variation risks. The fixed variation risk for Taiwan stock market prices is the lowest ($\alpha_{10} = 0.0395$). The square item of the Canada stock market returns also affects the variation risk of these three stock markets. An impact affect for Taiwan stock market returns is the lowest ($\eta_1 = 0.0429$). Moreover, $\alpha_{11} + \beta_{11} + \eta_1 = 1$, $\alpha_{22} + \beta_{21} + \eta_2 = 1$, and $\alpha_{31} + \beta_{31} + \eta_3 = 1$ conforms to the parameters of the IGARCH model's conditional supposition.

Table 8. Parameter estimation of the trivariate IGARCH(1, 1) model

Parameter	ϕ_0	ϕ_{11}	ϕ_{21}	ϕ_{31}	ϕ_{41}
Coefficient	0.0507**	-0.0482*	-0.0820***	0.0306	0.4322***
(p-value)	(0.0394)	(0.0995)	(0.0002)	(0.3179)	(0.0000)
Parameter	φ_0	φ_{11}	φ_{21}	φ_{31}	φ_{41}
Coefficient	0.1021***	-0.0112	-0.0498	0.0315	0.3997***
(p-value)	(0.0006)	(0.7439)	(0.1131)	(0.4168)	(0.0000)
Parameter	ψ_0	ψ_{11}	ψ_{21}	ψ_{31}	ψ_{41}
Coefficient	0.0559**	-0.0353	-0.0387**	-0.0139	0.3028***
(p-value)	(0.0114)	(0.1799)	(0.0474)	(0.6925)	(0.0000)
Parameter	α_{10}	α_{11}	β_{11}	η_1	α_{20}

Coefficient	0.0272***	0.0644***	0.9046***	0.0310***	0.1012***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	α_{21}	β_{21}	η_2	α_{30}	α_{31}
Coefficient	0.1347***	0.8105***	0.0548***	0.0386***	0.1442***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	β_{31}	η_3			
Coefficient	0.8159***	0.0399***			
(p-value)	(0.0051)	(0.0000)			
Parameter	c_0	c_1	c_2	d_0	d_1
Coefficient	-1.2014***	2.5887***	0.0978***	-2.0669***	4.0910***
(p-value)	(0.0000)	(0.0000)	(0.0010)	(0.0000)	(0.0000)
Parameter	d_2	e_0	e_1	e_2	
Coefficient	0.0778***	-1.7305***	3.4376***	0.1136***	
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
Parameter	$\bar{\rho}_{12}$	$\bar{\rho}_{13}$	$\bar{\rho}_{23}$	min ρ_{12}	max ρ_{12}
Coefficient	0.6086***	0.6868***	0.5835***	0.4666	1.0000
(p-value)	(0.0000)	(0.0000)	(0.0000)		
Parameter	min ρ_{13}	max ρ_{13}	min ρ_{23}	max ρ_{23}	
Coefficient	0.3168	0.9979	0.3787	0.9990	
(p-value)					

Notes: (1) p-value < α denotes significance (α =1%, α =5%, α =10%).

(2) * denotes significance at the level 10%, ** denotes significance at the level 5%, and *** denotes significance at the level 1

4.2 Model Checking of the Standard Residual Error for the Trivariate IGARCH Model

The trivariate IGARCH model will be appropriate for examining the standard residual error and a standard residual error square series by the test method of Ljung-Box and determining whether they still have auto-correlation. This is done by the standard residual error Q test of LB (5) to LB (25) with a P-value and the standard residual error square series Q test of LB^2 (5) to LB^2 (25) with a P-value in Table 9. The diagnosis finds that the trivariate GARCH(1, 2) model already does not have an auto-correlation with the standard residual error. This is also shown by Table 10. The trivariate IGARCH(1, 1) model already does not have an ARCH effect on the standard residual error square series. Therefore, the proposed model is suitably matched and is appropriate for these purposes.

Table 9. Q test of the standard residual error and its squared series

Taiwan	$LB(5)$	$LB(10)$	$LB(15)$	$LB(20)$	$LB(25)$
Q statistic	1.1704	7.2096	17.0979	21.4448	22.4637
(p-value)	(0.9477)	(0.7055)	(0.3130)	(0.3714)	(0.6088)
Taiwan	$LB^2(5)$	$LB^2(10)$	$LB^2(15)$	$LB^2(20)$	$LB^2(25)$
Q statistic	6.0926	11.5152	18.2379	26.3653	27.8861
(p-value)	(0.2973)	(0.3188)	(0.2504)	(0.1541)	(0.3131)
Korea	$LB(5)$	$LB(10)$	$LB(15)$	$LB(20)$	$LB(25)$
Q statistic	2.4038	7.7894	12.5184	15.3920	18.3726
(p-value)	(0.7909)	(0.6494)	(0.6394)	(0.7536)	(0.8262)
Korea	$LB^2(5)$	$LB^2(10)$	$LB^2(15)$	$LB^2(20)$	$LB^2(25)$
Q statistic	9.7417	11.0597	19.0273	24.3810	26.2697
(p-value)	(0.0829)	(0.3529)	(0.2125)	(0.2261)	(0.3933)
Singapore	$LB(5)$	$LB(10)$	$LB(15)$	$LB(20)$	$LB(25)$
Q statistic	0.9002	6.6387	11.7818	14.4956	15.5444
(p-value)	(0.9702)	(0.7591)	(0.6955)	(0.8045)	(0.9277)
Singapore	$LB^2(5)$	$LB^2(10)$	$LB^2(15)$	$LB^2(20)$	$LB^2(25)$
Q statistic	6.3633	7.9671	16.3255	19.0896	23.4410
(p-value)	(0.2725)	(0.6320)	(0.3608)	(0.5160)	(0.5518)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

Table 10. L-B test-ARCH effect test of the standard residual error

Taiwan	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	-1.3610	-1.5598	-1.4636	Statistic	1.1059
(p-value)	(0.1736)	(0.1189)	(0.1434)	(p-value)	(0.3164)
Korea	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	-0.8520	-0.4782	-0.7971	Statistic	0.9245
(p-value)	(0.3943)	(0.6326)	(0.4255)	(p-value)	(0.5844)
Singapore	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	0.3002	-0.3612	-0.2971	Statistic	0.8630
(p-value)	(0.7641)	(0.7180)	(0.7664)	(p-value)	(0.6801)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

5. Asymmetric Test of the Trivariate IGARCH(1, 1) Model

Because of the parameter estimation and the standard residual error diagnosis in the above IGARCH(1, 1) model, the examination can only determine if the model matches the suitable quality, and cannot determine whether the model has an asymmetrical phenomenon. Therefore, Engle and Ng (1993) developed a diagnosis test to examine whether or not the model has asymmetrical risk, and we use this diagnostic test to carry out the examination.

Engle and Ng (1993) considered that by observing the variables' past value, it is possible to forecast the standardized residual error square $(a_t/\sigma_t)^2$, $\sigma_t = (h_t)^{1/2}$. However, if there is no forecast pattern of the variables' past value, then the expression model may be set up incorrectly. Therefore, there are four examination methods for the model hypotheses:

(1) Sign Bias Test

$$(a_t/\sigma_t)^2 = b_0 + b_1 S_{t-1}^- + e_t, \tag{19}$$

(2) Negative Size Bias Test

$$(a_t/\sigma_t)^2 = b_0 + b_1 S_{t-1}^-(a_{t-1}/\sigma_{t-1}) + e_t, \tag{20}$$

(3) Positive Size Bias Test

$$(a_t/\sigma_t)^2 = b_0 + b_1(1 - S_{t-1}^-)(a_{t-1}/\sigma_{t-1}) + e_t, \tag{21}$$

(4) Joint Test

$$(a_t/\sigma_t)^2 = b_0 + b_1 S_{t-1}^- + b_2 S_{t-1}^-(a_{t-1}/\sigma_{t-1}) + b_3(1 - S_{t-1}^-)(a_{t-1}/\sigma_{t-1}) + e_t, \tag{22}$$

where S_{t-1}^- is the dummy variable: as $a_t \le 0$, then S_t^-=1; as a_t >0, then S_t^-=0.

After the above-mentioned results, Table 11 asymmetrically examines the results for Taiwan stock market prices, indicating that: (a) The sign bias test does not reveal (α=10%). (b) The negative size bias test does not reveal (α=10%). (c) The positive size bias test reveals (α=10%). (d) The joint test does not reveal (α=10%). Table 11 asymmetrically examines the results for Korea stock market prices, indicating that: (a) The sign bias test does not reveal (α=10%). (b) The negative size bias test does not reveal (α=10%). (c) The positive size bias test does not reveal (α=10%). (d) The joint test does not reveal (α=10%). Table 11 asymmetrically examines the results for Singapore stock market prices, indicating that: (a) The sign bias test dos not reveal (α=10%). (b) The negative size bias test does not reveal (α=10%). (c) The positive size bias test does not reveal (α=10%). (d) The joint test does not reveal (α=10%). From the joint test, it is shown that an asymmetrical phenomenon do not exist among the of the Taiwan, Singapore and KOREA stock market prices during the sample period.

Table 11. Asymmetric test of the trivariate IGARCH model

Taiwan	Sign bias test	Negative size Bias test	Positive size Bias test	Joint test
F statistic (p-value)	2.1756 (0.1404)	1.7011 (0.1923)	5.1968** (0.0227)	2.5639* (0.0531)
Korea	Sign bias test	Negative size Bias test	Positive size Bias test	Joint test
F statistic (p-value)	3.8638** (0.0495)	3.1632* (0.0755)	6.5360** (0.0106)	3.8717*** (0.0089)
Singapore	Sign bias test	Negative size Bias test	Positive size Bias test	Joint test
F statistic (p-value)	0.1247 (0.7240)	0.7288 (0.3934)	0.1572 (0.6918)	0.2907 (0.8322)

Notes: p-value< α denotes significance (α =1%, α =5%, α =10%).

* denotes significance at the level 10%, ** denotes significance at the level 5%, and

*** denotes significance at the level 1%.

Table 11-1. **Granger Causality** test of the Taiwan, Korea, Singapore and Canada stock markets with lag is equals 1.

Null Hypothesis	F-Statistic	p-value
CANA does not Granger Cause Taiwan	13.3527	0.00026
CANA does not Granger Cause KOREA	6.73334	0.00953
CANA does not Granger Cause SING	7.16262	0.00750

Table 11-2. **Granger Causality** test of the Taiwan, Korea, Singapore and Canada stock markets with lag is equals 5.

Null Hypothesis	F-Statistic	p-value
CANA does not Granger Cause Taiwan	71.2055	3.9E-69
CANA does not Granger Cause KOREA	47.3913	1.3E-46
CANA does not Granger Cause SING	49.4701	1.3E-48

6. Trivariate asymmetric GARCH Model

6.1 Trivariate asymmetric GARCH model and parameter estimation

This section uses the trivariate asymmetric GARCH model- namely equations uses (3)-(18) and the idea of GJR-GARCH model to analyze relatedness of the Taiwan, the Korea, and the Singapore stock price return volatilities. In this paper also further considers that the variation risk of the Taiwan, the Korea and the Singapore stock markets whether receives the affect of the the square item of the Canada stock market eturns. The parameter estimation first considers a general model and is based on the estimated results. Then we then deletes some explanation variables that are not significant here. Finally, we also obtains a simplification model for the Taiwan, the Korea, and the Singapore stock price return volatility relatedness analysis. The empirical results show that the Taiwan, the Korea and the Singapore stock price return volatilities may be constructed on the trivariate asymmetric IGARCH(1, 1) model. Its estimate results are also stated in Table 12. The proposed model is given as follows:

$$RTW_t = \phi_0 + \phi_{11} RTW_{t-1} + \phi_{21} RKOREA_{t-1} + \phi_{31} RSING_{t-1} + \phi_{41} RCANA_{t-1} + a_{1,t} \tag{23}$$

$$RKOREA_t = \varphi_0 + \varphi_{11} RTW_{t-1} + \varphi_{21} RKOREA_{t-1} + \varphi_{31} RSING_{t-1} + \varphi_{41} RCANA_{t-1} + a_{2,t} \tag{24}$$

$$RSING_t = \psi_0 + \psi_{11} RTW_{t-1} + \psi_{21} RKOREA_{t-1} + \psi_{31} RSING_{t-1} + \psi_{41} RCANA_{t-1} + a_{3,t} \tag{25}$$

$$\begin{aligned} h_{11,t} = {} & u_{t-1} \times (\alpha_{10} + \alpha_{11} a_{1,t-1}^2 + \beta_{11} h_{11,t-1}) + \\ & (1-u_{t-1}) \times (\alpha'_{10} + \alpha'_{11} a_{1,t-1}^2 + \beta'_{11} h_{11,t-1}) \end{aligned} \tag{26}$$

$$\begin{aligned} h_{22,t} = {} & u_{t-1} \times (\alpha_{20} + \alpha_{21} a_{2,t-1}^2 + \beta_{21} h_{22,t-1}) + \\ & (1-u_{t-1}) \times (\alpha'_{20} + \alpha'_{21} a_{2,t-1}^2 + \beta'_{21} h_{22,t-1}) \end{aligned} \tag{27}$$

$$\begin{aligned} h_{33,t} = {} & u_{t-1} \times (\alpha_{30} + \alpha_{31} a_{3,t-1}^2 + \beta_{31} h_{33,t-1}) + \\ & (1-u_{t-1}) \times (\alpha'_{30} + \alpha_{31} a_{3,t-1}^2 + \beta'_{31} h_{33,t-1}) \end{aligned} \tag{28}$$

$$q_{12,t} = c_0 + c_1 \rho_{12,t-1} + c_2 a_{1,t-1} a_{2,t-1} / \sqrt{h_{11,t-1} h_{22,t-1}} \tag{29}$$

$$q_{13,t} = d_0 + d_1 \rho_{13,t-1} + d_2 a_{1,t-1} a_{3,t-1} / \sqrt{h_{11,t-1} h_{33,t-1}} \tag{30}$$

$$q_{23,t} = e_0 + e_1 \rho_{23,t-1} + e_2 a_{2,t-1} a_{3,t-1} / \sqrt{h_{22,t-1} h_{33,t-1}} \tag{31}$$

$$\rho_{12,t} = \exp(q_{12,t}) / (\exp(q_{12,t}) + 1) \tag{32}$$

$$\rho_{13,t} = \exp(q_{13,t}) / (\exp(q_{13,t}) + 1) \tag{33}$$

$$\rho_{23,t} = \exp(q_{23,t}) / (\exp(q_{23,t}) + 1) \tag{34}$$

$$h_{12,t} = \rho_{12} \sqrt{h_{11,t}} \sqrt{h_{22,t}} \ , \ h_{13,t} = \rho_{13} \sqrt{h_{11,t}} \sqrt{h_{33,t}} \ , \ h_{23,t} = \rho_{23} \sqrt{h_{22,t}} \sqrt{h_{33,t}} \tag{35}$$

$$u_t = \begin{cases} 1 \ , & if \ \ RCANA_t \le 0 \\ 0 & if \ \ RCANA_t > 0 \end{cases} \tag{36}$$

where $\vec{a}'_t = (a_{1,t}, a_{2,t}, a_{3,t})$ obeys the trivariate normal distribution- namely, $N(\vec{0}, H_t)$, among

$\vec{0}' = (0,0,0)$ and

$$H_t = \begin{bmatrix} h_{11,t} & h_{21,t} & h_{31,t} \\ h_{12,t} & h_{22,t} & h_{32,t} \\ h_{13,t} & h_{23,t} & h_{33,t} \end{bmatrix}, \quad h_{12,t} = h_{21,t}, \quad h_{13,t} = h_{31,t}, \quad h_{23,t} = h_{32,t}.$$

The probability density function of $\vec{a}_t$ is

$$f(a_{1,t}, a_{2,t}, a_{3,t} \mid H_t) = \frac{1}{(2\pi)^{3/2}\left|H_t\right|^{1/2}} \exp\left\{\frac{-1}{2}\vec{a}_t' H_t^{-1} \vec{a}_t\right\} \tag{37}$$

Where $\rho_{12,t}$ is the dynamic conditional correlation (DCC) coefficient of $a_{1,t}$ and $a_{2,t}$, $\rho_{13,t}$ is the DCC coefficient of $a_{1,t}$ and $a_{3,t}$, and $\rho_{23,t}$ is the DCC coefficient of $a_{2,t}$ and $a_{3,t}$. In addition, H_t^{-1} is the inverse matrix of H_t. In this paper, we use the normal distribution for the stochastic error term, and also use the maximum likelihood algorithm method of BHHH (Berndt et. al., 1974) to estimate the parameters of the proposed model.

Based on the estimated results of the trivariate asymmetric GARCH(1, 1) model in Table 12, we test whether or not the estimated value of the parameters' coefficient is significant with a P-value. In the sample period, the Taiwan stock price return does not receive the affected of the constant item. The Taiwan stock price return receives previous one periods' influence of the Hong Kong stock returns. The Taiwan stock price return receives the previous one periods' influence of the Korea and the Canada stock price returns. The Taiwan stock price return does not receive the previous one periods' influence of the Singapore stock price returns. The Korea stock price return receives the affected of the constant term. The Korea stock price return receives the previous one periods' affected of the Korea and the Canada stock price returns. The Korea stock price return does not receive the previous one periods' affected of the Taiwan and the Singapore stock price returns. Similarly, the Singapore stock price return also receives the affected of the constant term. The Singapore stock price return receives the previous one periods' affected of the Taiwan, the Korea and the Canada stock price returns.

On the other hand, the correlation coefficient value of the Taiwan and the Korea stock price return volatilities is significant ($\bar{\rho}_{12}$=0.6724). This result means that the Korea stock price return's volatility is a positive influence to the Taiwan stock price return's volatility, and they have precisely synchronized mutual influence. When the variation risk of the Korea stock price return increase, the investor's risk in the Taiwan stock price market can increase. Likewise, when the variation risk of the Korea stock price return decrease, the investor's risk in the Taiwan stock price market is also can decrease. Similarly, the correlation coefficient value of the Taiwan and the Singapore stock price return volatilities is significant ($\bar{\rho}_{13}$=0.5952). This result also shows that Taiwan stock price return's volatility is a positive influence to the Singapore stock price return's volatility. The correlation coefficient value of the Korea and the Singapore stock price return volatilities is also significant ($\bar{\rho}_{23}$=0.5920).

This result also shows that the Korea stock price return's volatility is a positive influence to the Singapore stock price return's volatility, and the three stock price markets has a high relation.

The observed conditional variance equation of the estimated coefficient, under the 5% significance level, demonstrates that all the conditional variance estimated coefficients are significant in Table 12. The empirical results show that the conditional variance produces the different variation risks under good news and bad news. The previous one periods' residual error square item and the previous two period's conditional variance will affect the Taiwan, the Korea, and the Singapore stock price return rates' volatility. Under the bad news and good news ($CANA_{1t} \leq 0$ and $CANA_{1t} > 0$, respectively) for the Taiwan stock market returns, we have $\alpha_{11} + \beta_{11} = 1$ and $\alpha'_{11} + \beta'_{11} = 1$. The square item of the Canada stock return affects the variation risk of the Taiwan stock market. Under the bad news and good news for the Korea stock market returns, we have $\alpha_{21} + \beta_{21} = 1$ and $\alpha'_{21} + \beta'_{21} = 1$. The square item of the Canada stock return affects the variation risk of the Singapore stock market. Under the Singapore stock market, we have $\alpha_{31} + \beta_{31} = 1$ and $\alpha'_{31} + \beta'_{31} = 1$. Those three stock markets conforms to the parameter of the IGARCH model's conditional supposition. The empirical result shows that the good news and bad news of the Canada stock market returns will actually affect the variation risks of those three stock market returns. The traditional trivariate GARCH model can not respond to this information, but the trivariate asymmetric GARCH(1, 1) model may find the Taiwan, the Korea and Singapore stock price return rates' volatility process. The likelihood ratio test is also supported the trivariate asymmetric GARCH(1, 1) model. Therefore, the explanatory ability of the trivariate asymmetric IGARCH(1, 1) model is better than the traditional model of the trivariate GARCH.

Table 12. Parameters' estimation of the trivariate asymmetric IGARCH(1, 1) model

Parameter	ϕ_0	ϕ_{11}	ϕ_{21}	ϕ_{31}	ϕ_{41}
Coefficient	0.0500**	-0.0388	-0.0293	0.0938***	0.3292***
(p-value)	(0.0188)	(0.1584)	(0.2027)	(0.0005)	(0.0000)
Parameter	φ_0	φ_{11}	φ_{21}	φ_{31}	φ_{41}
Coefficient	0.0731***	-0.0458*	-0.0765***	0.0748**	0.4088***
(p-value)	(0.0009)	(0.0966)	(0.0072)	(0.0100)	(0.0000)
Parameter	ψ_0	ψ_{11}	ψ_{21}	ψ_{31}	ψ_{41}
Coefficient	0.0494***	-0.0358*	-0.0389**	-0.0241	0.3205***
(p-value)	(0.0034)	(0.0773)	(0.0359)	(0.3535)	(0.0000)
Parameter	α_{10}	α_{11}	β_{11}	α'_{10}	α'_{11}
Coefficient	0.0707***	0.0870***	0.9130***	0.0000	0.1347***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(1.0000)	(0.0000)

Parameter	β'_{11}	α_{20}	α_{21}	β_{21}	α'_{20}
Coefficient	0.8653***	0.0986***	0.1621***	0.8379***	0.0000
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(1.0000)
Parameter	α'_{21}	β'_{21}	α_{30}	α_{31}	β_{31}
Coefficient	0.1382***	0.8618***	0.0397***	0.1253***	0.8747***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	α'_{30}	α'_{31}	β'_{31}		
Coefficient	0.0000	0.1295***	0.8705***		
(p-value)	(1.0000)	(0.0000)	(0.0000)		
Parameter	c_0	c_1	c_2	d_0	d_1
Coefficient	-2.0597***	4.0614***	0.1036***	-1.8673***	3.7213***
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Parameter	d_2	e_0	e_1	e_2	
Coefficient	0.0790***	-1.9469***	3.8435***	0.1016***	
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
Parameter	$\bar{\rho}_{12}$	$\bar{\rho}_{13}$	$\bar{\rho}_{23}$	min ρ_{12}	max ρ_{12}
Coefficient	0.6724***	0.5952***	0.5920***	0.4645	0.9998
(p-value)	(0.0000)	(0.0000)	(0.0000)		
Parameter	min ρ_{13}	max ρ_{13}	min ρ_{23}	max ρ_{23}	
Coefficient	0.2700	0.9821	0.2665	0.9982	
(p-value)					

Notes: p-value < α denotes significance (α =1%, α =5%, α =10%).

6.2 Model checking of the trivariate asymmetric IGARCH(1, 1)

Correct the inappropriateness of the trivariate asymmetric IGARCH model, the test method of Ljung-Box is used to further examine the standard residual error and a standard residual error square item to see whether auto-correlation still exists. Table 13 shows that the Q test of the standard residual error and the standard residual error square item with a P-value. Clearly, this model does not have auto-correlation. Table 14 shows that the trivariate asymmetric IGARCH(1, 1) model does not have the ARCH effect of the standard residual error square item. Therefore, proposed model matches quite suitably and is more appropriate to use.

Table 13. L-B Q test of the standard residual error and its squared series

Taiwan	LB (5)	LB (10)	LB (15)	LB (20)	LB (25)
Q statistic	1.1704	7.2096	17.0979	21.4448	22.4637
(p-value)	(0.9477)	(0.7055)	(0.3130)	(0.3714)	(0.6088)

Taiwan	$LB^2(5)$	$LB^2(10)$	$LB^2(15)$	$LB^2(20)$	$LB^2(25)$
Q statistic	6.0926	11.5152	18.2379	26.3653	27.8861
(p-value)	(0.2973)	(0.3188)	(0.2504)	(0.1541)	(0.3131)
Korea	$LB(5)$	$LB(10)$	$LB(15)$	$LB(20)$	$LB(25)$
Q statistic	2.4038	7.7894	12.5184	15.3920	18.3726
(p-value)	(0.7909)	(0.6494)	(0.6394)	(0.7536)	(0.8262)
Korea	$LB^2(5)$	$LB^2(10)$	$LB^2(15)$	$LB^2(20)$	$LB^2(25)$
Q statistic	9.7417	11.0597	19.0273	24.3810	26.2697
(p-value)	(0.0829)	(0.3529)	(0.2125)	(0.2261)	(0.3933)
Singapore	LB **(5)**	LB **(10)**	LB **(15)**	LB **(20)**	LB **(25)**
Q statistic	0.9002	6.6387	11.7818	14.4956	15.5444
(p-value)	(0.9702)	(0.7591)	(0.6955)	(0.8045)	(0.9277)
Singapore	LB^2 **(5)**	LB^2 **(10)**	LB^2 **(15)**	LB^2 **(20)**	LB^2 **(25)**
Q statistic	6.3633	7.9671	16.3255	19.0896	23.4410
(p-value)	(0.2725)	(0.6320)	(0.3608)	(0.5160)	(0.5518)

Notes: p-value< α denotes significance (α =5%).

Table 14. L-B test- ARCH effect test of the standard residual error

Taiwan	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	-1.3610	-1.5598	-1.4636	Statistic	1.1059
(p-value)	(0.1736)	(0.1189)	(0.1434)	(p-value)	(0.3164)
Korea	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	-0.8520	-0.4782	-0.7971	Statistic	0.9245
(p-value)	(0.3943)	(0.6326)	(0.4255)	(p-value)	(0.5844)
Singapore	$LB^2(10)$	$LB^2(20)$	$LB^2(30)$	F test	
Q statistic	0.3002	-0.3612	-0.2971	Statistic	0.8630
(p-value)	(0.7641)	(0.7180)	(0.7664)	(p-value)	(0.6801)

Notes: p-value< α denotes significance (α =5%).

7. Conclusions and Recommendations

There are many factors that may influence stock prices, including overall economic agents, overall currency supplies, interest rates, prices, and inflation rates. Each factor can influence stock price returns. This study has discussed three stock market return volatilities that influence the Taiwan, Korea, and Singapore's markets. We have use data from January 2003 to December 2013 on the Taiwan weight stock, the Singapore Strait Times, the Korea KOSPI, and the Torondo 300 stock indices as the sample. The empirical results show that

Taiwan, Singapore, and Korea stock price return volatilities may be constructed in the trivariate normal distribution and the asymmetric IGARCH(1, 1) model. This model also passes a standard residual error and ARCH effect tests. This demonstrates that the trivariate IGARCH(1, 1) model is a more appropriate fit. The empirical results also show that the correlation coefficient value ($\overline{\rho}_{12}$=0.6724) for Taiwan and Korea stock market returns has positive relations, the correlation coefficient value ($\overline{\rho}_{13}$=0.5952) for Taiwan and Singapore market returns has positive relations, and the correlation coefficient value ($\overline{\rho}_{23}$=0.5920) for the Korea and Singapore returns also has positive relations. This result demonstrates that Korea stock return volatility affects Taiwan and Singapore stock returns' variation risk, and Taiwan stock return volatility does not affect Korea stock returns' variation risk. The empirical results also show that Taiwan and Korea stock market returns' volatilities have the asymmetrical phenomenon during the sample period. Good news and bad news actually affect the variation risks of the stock market returns.

Empirical results show that Taiwan, Singapore, and Korea stock price return volatilities have mutual relationships. This evidence demonstrates that Korea stock returns' risk can affect the risk to Taiwan and Singapore stock market returns. If investors can clearly understand the process of stock market return volatilities, it will be easier for them to evaluate the three stock markets and their investment returns and to determine an investment strategy.

The empirical results also suggest that domestic or overseas investors carrying out investment decision-making for the Taiwan, Singapore, and Korea stock markets can consider only the targeted stock market at that time in order to judge the investment profits, or consider only the stock market's own return and own risk, while they can neglect the return risks and volatility properties of the other stock markets. Nevertheless, this may result in the overestimation or underestimation of investment returns, and will also produce uncertainty about return on investment. Therefore, when regulatory authorities attempt to stabilize a stock market, if at the same time the other area's stock market stability can be further appraised, then the intended policy goal can be easily achieved.

However, there are many theories and models for the returns and the volatility properties of financial commodities. This research uses only the trivariate asymmetric GARCH model to discuss the three stock markets of Taiwan, Singapore, and Korea. For future research, we suggest that other asymmetries of the multivariate GARCH model or other models can be used for further analysis.

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