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An Investigation into the Multifractal Characteristics of the TAIEX Stock Exchange Index in Taiwan

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This paper analyzes the minute-by-minute variations of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) over eight years by using the box-counting multifractal spectum $f(\alpha)$. The results reveal that the daily return R is directly correlated with the absolute value of $\Delta\alpha$ for that day while a positive or negative sign of Δf is related to an increasing or decreasing return, respectively The gain probability ($G\%$) and the index increase probability ($N\%$) attain 65 $\sim$ 74 % when Δf has a positive value and 8 $\sim$ 32 % when Δf has a negative value, but both converge toward 50 % with the number of days considered when computing the value of Δf increases. With regard to prediction of the future index movement the results show that the sign sequences of Δf provide a more reliable predictive performance than those of the index variation parameter ΔI. The correlation between the risk measurement parameter R_f and the increasing or decreasing tendency of the TAIEX price index is also examined in this paper and results are opposite to those presented for the SSEC index in China thus suggesting that the phenomenon is market dependent.

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I. INTRODUCTION

It is commonly maintained that stock markets exhibit a random walk characteristic and that past price alone therefore provides an unreliable indication of the future price movement. However, recent empirical studies have reported that the price variation is not in fact totally unpredictable. For example, Sun *et al.* [1,2] utilized a multifractal approach to analyze the minute-by-minute Hang Seng index data of the Hong Kong stock market over the period from January 3 1994 to May 3 1997 (a total of 838 trading days). The authors calculated the multifractal spectrum (*i.e.* the $f(\alpha)$ curve) of the daily return data and then applied statistical analysis techniques to determine whether or not the multifractal spectrum parameters, *i.e.* $\Delta\alpha$ and Δf, were correlated with the variation in the closing return R. The empirical results revealed that the magnitude of the variation in R was directly correlated with the value of $\Delta\alpha$ for that day. Furthermore, it was shown that an increasing or decreasing tendency of the return was directly related to a positive or negative value of Δf, respectively. The authors also studied the dependencies of the daily gain probability ($G\%$) and the index increase probability ($N\%$) on the value of Δf, where Δf was computed by aggregating the individual daily values of Δf over the previous several days and the results showed that a strong correlation between the return variation and the value of Δf was maintained for 1 – 3 days.

In a more recent study, the same group demonstrated that the sequence of positive or negative sign of Δf provided more accurate predictions of the Hang Seng index movement than the ΔI (closing index variation) [3]. Wei and Huang [4] analyzed the 5-minute returns of the Shanghai Stock Exchange Composite (SSEC) index over the period extending from January 19 1999 to July 6 2001 (a total of 586 trading days) and found that in contrast to the Hang Seng index, the value of Δf decreased rather than increased with an increasing daily return. Therefore, they concluded that the correlation between Δf and the daily return was stock market dependent. Accordingly, they proposed a new market risk measurement index, R_f, based upon both $\Delta\alpha$ and Δf and showed that R_f was more strongly correlated with the daily return of the SSEC index than Δf. Furthermore, they demonstrated that the value of R_f for the current day could be used to predict the values of the gain probability ($G\%$) and the index increase probability ($N\%$) parameters for the following day.

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In the regional economy in Asia, many researchers have used some tools in econophysics to analyze the financial markets in their own countries, such as Mainland China [4–7], Hong Kong [8–10] and Korea [11–18]. However, compared to the financial markets in Mainland China and Hong Kong, the stock market in Taiwan has received comparatively little attention. In the past, only Ho *et al.* [19] and Di Matteo *et al.* [20, 21] have ever used tools to analyze the daily Taiwan stock price index. However, after all the daily time series still reveals less temporal structure information than the high-frequency return [22]. Accordingly, the present study employs a multifractal approach to analyze the minute-by-minute return data of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) over the period between May 3 1999 and November 30 2007. The results are then compared with those presented in the literature for the Hang Seng index in Hong Kong, the SSEC index in Mainland China and the NYSE index in the USA. Having removed weekends and holidays from the considered timeframe, a total of 2162 trading days remained. Between May 3 1999 and December 30 2000, Taiwan's stock market traded between the hours of 9:00 am and 12:00 pm; *i.e.* the timeframe includes a total of 453 trading days with a trading time of 180 minutes on each day. Between January 1 2001 and November 30 2007, trading was extended to 13:30 pm; *i.e.* the timeframe included a total of 1709 trading days with a daily trading time of 270 minutes. As a result, the total amount of minute-by-minute data available for analysis purposes is given by 453 (days) × 180 (minutes) + 1709 (days) × 270 (minutes) = 542,970 data points.

The remainder of this paper is organized as follows: Section II introduces the basic concepts and parameters of the multifractal spectra used in this study to analyze the TAIEX data. Section III begins by investigating the correlation between Δf and the daily TAIEX return. The dependencies of the daily gain probability ($G\%$) and index increase probability ($N\%$) parameters on Δf are then systematically examined. The section concludes by examining the feasibility of predicting future movements of the TAIEX index by using the market risk index R_f or the sign sequences of Δf and ΔI, respectively. Finally, Section IV summarizes the major findings and contributions of the study.

II. RESEARCH METHOD

In the current analysis, the TAIEX price at time t is denoted as $I(t)$, where t has units of one minute and has a value in the range $1 \sim 542970$. As in Refs. 1-4, the multifractal spectra used in this study to analyze the TAIEX data are calculated using the box-counting method, in which the index variation for each day is covered with multiple boxes (*i.e.* multiple time intervals). Assuming the size of each covering box is denoted as l, the integer number of boxes required to cover the index variation is equal to the trading time for that day divided by an appropriate value of l. For example, if the trading time is 270 minutes, l can be specified as 1, 2, 3, 5, 6, 9, 15, 18, 27, 30, 45, 54, 90, 135 or 270 minutes, respectively. Similarly, for the case of a trading time of 180 minutes, l can be specified as 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90 or 180 minutes. Let $P_i(l)$ denote the proportion of the entire index on a single day that falls within the i^{th}-box. Assuming a trading time of 180 minutes and a box size of 30 minutes, $P_i(l)$ is given by

$$P_i(l) = \sum_{t=(i-1)*30+1}^{t=i*30} I(t) / \sum_{t=1}^{t=180} I(t), \quad i = 1, 2, 3, \cdots, 6 \tag{1}$$

where $I(t)$ is the TAIEX index at time t In the limiting case of $l \to 0$, $P_i(l)$ can be defined as where $I(t)$ is the TAIEX index at time t In the limiting case of $l \to 0$, $P_i(l)$ can be defined as

$$P_i(l) \sim l^{\alpha}, \tag{2}$$

where the exponent α represents the singularity strength (or Hölder exponent) of the probability measure. Counting the number of boxes $N(\alpha)$ for which the probability measure P_i has a singularity strength between α and $\alpha + d\alpha$, we can broadly defined $f(\alpha)$ as the fractal dimension of the set of boxes having a singularity strength α [23], *i.e.*

$$N(\alpha) \sim l^{-f(\alpha)}. \tag{3}$$

Eq. (3) thus describes a multifractal measure in terms of interwoven sets of different singularity strengths α, each characterized by its own fractal dimension $f(\alpha)$.

Another method to calculate α and $f(\alpha)$ is to use the partition function $\chi_q(l)$ defined as

$$\chi_q(l) = \sum_{i=1}^{m} P_i^q(l) \sim l^{\tau(q)}, \tag{4}$$

where the probability measure P_i is raised to a power of q and m is the total number of boxes used to cover the index variation. Note that $\chi_q(l)$ with $q \to +\infty$ is associated with the largest probability regions in the set while $\chi_q(l)$ with $q \to -\infty$ is associated with the smallest probability regions in the set. In the present multifractal calculations, the maximum value of $|q|$ is 120. The value of $\tau(q)$ in Eq. (4) can be obtained from the slope of the linear region of the $\ln \chi_q(l) - \ln l$ curve. The multifractal spectral parameter $f(\alpha)$ can then be obtained by performing the following Legendre transformation [23]:

$$f(\alpha) = q\alpha - \tau(q), \quad \alpha = d\tau(q)/dq. \tag{5}$$

Multifractal spectra have two basic parameters, namely $\Delta\alpha$ and Δf. The first parameter denotes the width of the multifractal spectrum and is given by

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min}, \tag{6}$$

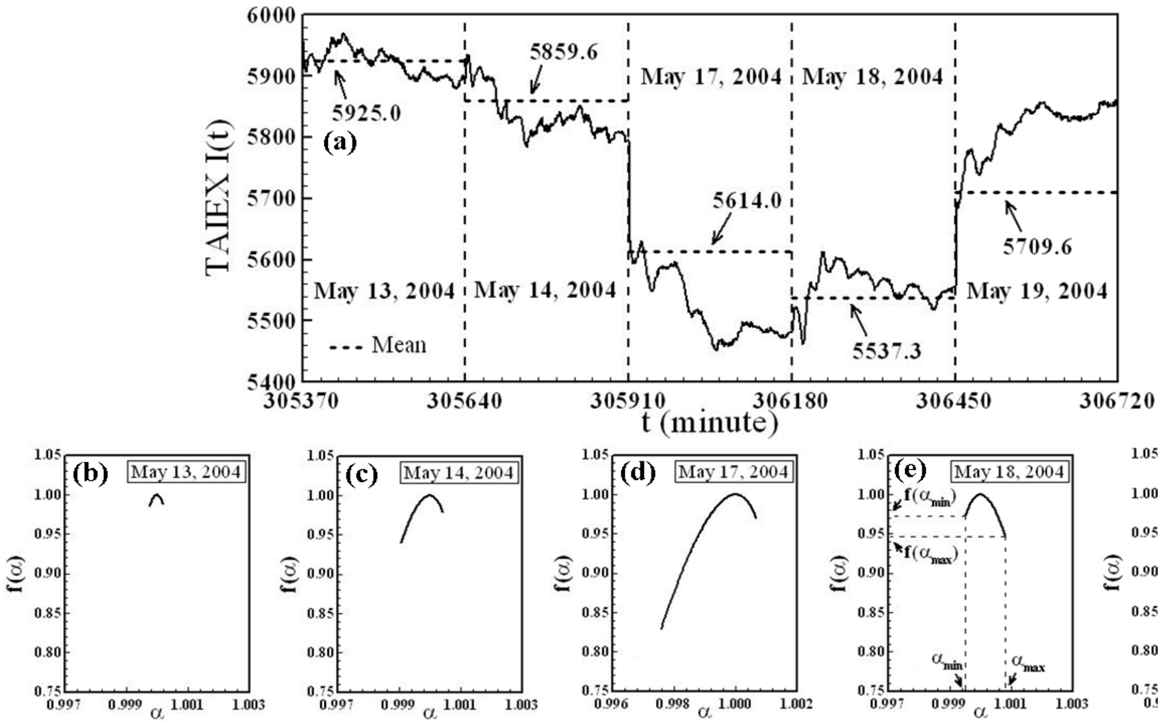

Fig. 1. (a) Variation of the minute-by-minute TAIEX index on May 13, 14, 17, 18 and 19 2004 (Note May 15 and 16 were weekend days and are therefore excluded here) (b)∼(f) Multifractal spectra corresponding to the five days shown in upper panel.

In the current context, a larger value of $\Delta\alpha$ implies a greater price fluctuation over the course of the day. In other words, absolute value of $\Delta\alpha$ provides an indication of the price volatility in any given trading day [4].

The second parameter, Δf, is defined as

$$\Delta f = f(\alpha_{\min}) - f(\alpha_{\max}). \quad (7)$$

Δf provides an insight into the tendencies of the price movement, *i.e.* an increasing tendency or a decreasing tendency [4].

Figure 1(a) presents the variation of the TAIEX index over five successive trading days in May 2004 *i.e.* May 13, 14, 17, 18 and 19. Note that May 15 and 16 were weekend days; thus no index data exists. The horizontal dashed lines in the figure denote the average of the minimum and the maximum indices for the corresponding day. It can be seen that the index variations are noticeably different from one day to the next. To enable the daily variations to be analyzed in a quantitative manner, Figures 1(b)∼1(f) present the multifractal spectra of the five daily indices shown in Figure 1(a). An inverted, downward-opening parabolic shape is seen in every case, thus confirming that the TAIEX data has a multifractal structure. Figure 1(e) indicates the principal parameters of interest when analyzing the multifractal spectra, namely the positions of the maximum and the minimum Hölder exponents, *i.e.* $\alpha_{\max}$ and $\alpha_{\min}$, respectively and the corresponding fractal dimensions of the set of boxes with singularity strengths $\alpha_{\max}$ and $\alpha_{\min}$, *i.e.* $f(\alpha_{\max})$ and $f(\alpha_{\min})$, respectively. Observing Figures 1(b)∼1(f), it can be seen that the differences in the daily variation characteristics evident in Figure 1(a) result in multifractal spectra with obviously different widths and shapes. As shown in Figure 1(a), the greatest variations in the TAIEX return over the period May 13 ∼ May 19 2004 took place on May 17 and May 19. On May 17, it can be seen that the index fell and remained below the horizontal dashed lines for most of the day. The corresponding multifractal spectrum has a hooklike characteristic and slants to the right, as shown in Figure 1(d). By contrast, the index on May 19 increased progressively over the course of the day and remained above the horizontal dashed lines for most of the trading period. In this case, the corresponding multifractal spectrum again has a hook-like characteristic, but slants to the left rather than to the right, as shown in Figure 1(f).

Observing the multifractal spectra for the remaining three days, we see that the different values of $\Delta\alpha$ and Δf observed in each case reflect different variations in the characteristics of the TAIEX data on the different days. Figure 2(a) presents the variation of the minute-by-minute TAIEX data over the considered time frame of 2162 days. Figures 2(b) ∼ (d) show the variations of the standard deviation of these data and the variations of $\Delta\alpha$ and Δf, respectively, over the same time period. The results show that fluctuations in the standard deviation of the minute-by-minute data produce corresponding fluctuations in the values of $\Delta\alpha$ and Δf, respectively. We observed that the fluctuations in $\Delta\alpha$ were very similar to those in the standard deviation of the index. Therefore,

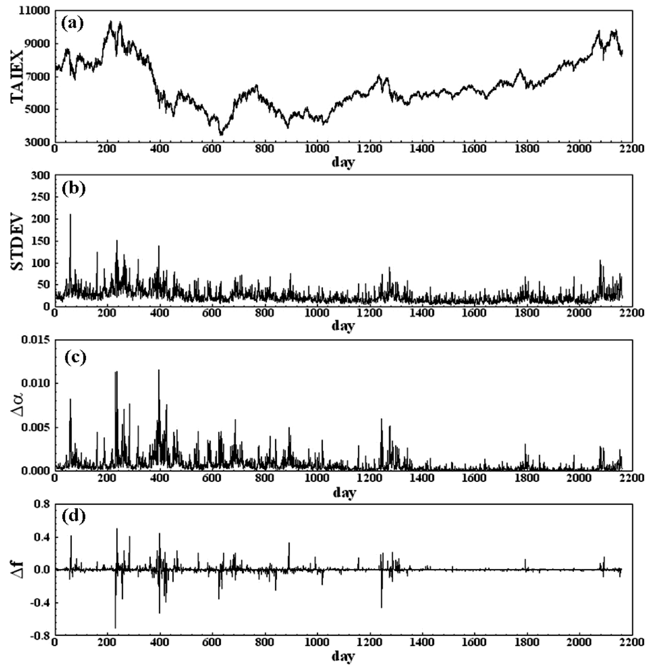

Fig. 2. (a) Variations of the minute-by-minute TAIEX index (b) The standard deviation of the TAIEX index (c) $\Delta\alpha$ of the TAIEX index and (d) Δf of TAIEX index over the period extending from May 3 1999 to November 30 2007 (a total of 2162 trading days).

we can be infer that the multifractal spectra shown in Figures 1(b) $\sim$ 1(f) contain meaningful statistical information regarding movements of the TAIEX price index.

III. RESULTS

Let the daily price fluctuation (*i.e.* the return) be defined as

$$R(t) = \ln I(t+\tau) - \ln I(t) = \ln\left[\frac{I(t+\tau)}{I(t)}\right], \quad (8)$$

where $\tau = 1$ day in the current case and $I(t)$ is the closing price. Figures 3(a) and 3(b) show the point distributions of $\Delta\alpha$ vs. R and Δf vs. R, respectively, for the TAIEX data between May 3 1999 and November 30 2007. Figure 3(a) shows that most of the data points are located near $R = 0$ and $\Delta\alpha = 0$. Off those points, which are located further from $R = 0$, however, it is evident that the value of $\Delta\alpha$ increases as the value of R deviates more significantly from zero in either the positive or the negative direction. These findings are consistent with those reported in Refs. 1 and 4 for the Hang Seng index in Hong Kong and the SSEC index in Mainland China. Figure 3(b) shows that more points are located in the first and the third quadrants than in the second and the fourth. The solid line indicates the least-squares fit of Δf as a function of R and runs from the bottom left of the figure to the top right. The positive slope of this line indicates a positive correlation between Δf and R; *i.e.* Δf increases with increasing R. Interestingly, this empirical result contradicts the findings presented in Ref. 4 for the SSEC index, but is consistent with the findings presented for the Hang Seng stock index in Ref. 1.

Wei and Huang [4] argued that traditional market risk measurements are based solely on the magnitude of the price fluctuations and therefore fail to consider the trends of price fluctuations. Therefore, they suggested a new multifractal-based risk measurement index,R_f, based not only on the absolute magnitude of the price fluctuations ($\Delta\alpha$), but also on the underlying tendency of these price fluctuations (Δf), *i.e.*

$$R_f(\tau) = \Delta\alpha(\tau)\mathrm{sign}(\Delta f(\tau))e^{|\Delta f(\tau)|}, \quad (9)$$

where $\tau = 1$ day and

$$\mathrm{sign}(\Delta f(\tau)) = \begin{cases} +1, & \text{when } \Delta f(\tau) > 0 \\ 0, & \text{when } \Delta f(\tau) = 0 \\ -1, & \text{when } \Delta f(\tau) < 0. \end{cases} \quad (10)$$

Figure 3(c) plots the variation of R_f with R for the current TAIEX data. Note that the two oblique solid and dashed lines in this figure represent least-squares fits of R_f as functions of R in the four quadrants. As for SSEC index in Shanghai Wei and Huang [4] argued that the different quadrants in this figure represent different economic scenarios. For example, quadrant (I), *i.e.* $R > 0$, $R_f > 0$, indicates that today's closing price is higher than yesterday's and the probability of today's price being above the mean price is greater than that of it being below. This condition implies that the price has been running at a high level for a relatively long time and, thus gives investors a clear "over-bought" signal. Consequently, investors will then sell their stocks and the index will decrease for this reason In quadrant (II), *i.e.* $R < 0$, $R_f > 0$, today's closing price is lower than that of the previous day and the probability of today's price being above the mean price is greater than that of it being below. On the one hand, these conditions imply that the price is decreasing and has a "weak" running tendency while on the other hand, they imply that the price has been running at a high level for a relatively long time, which indicates that the price has a "strong" running tendency. Thus, investors receive vague and ambiguous signals regarding the tendency of the index. As a result, the price is equally likely to increase or decrease on the following day. In quadrant (III), *i.e.* $R < 0$, $R_f > 0$, today's closing price is lower than that of the previous day and the probability of today's price being above the mean price is greater than that of it being below. On the one hand, these conditions imply that the price is decreasing and has a "weak" running tendency while on the other hand, they imply that the price has been running at a high level for a relatively long time, which indicates that the price has a "strong" running

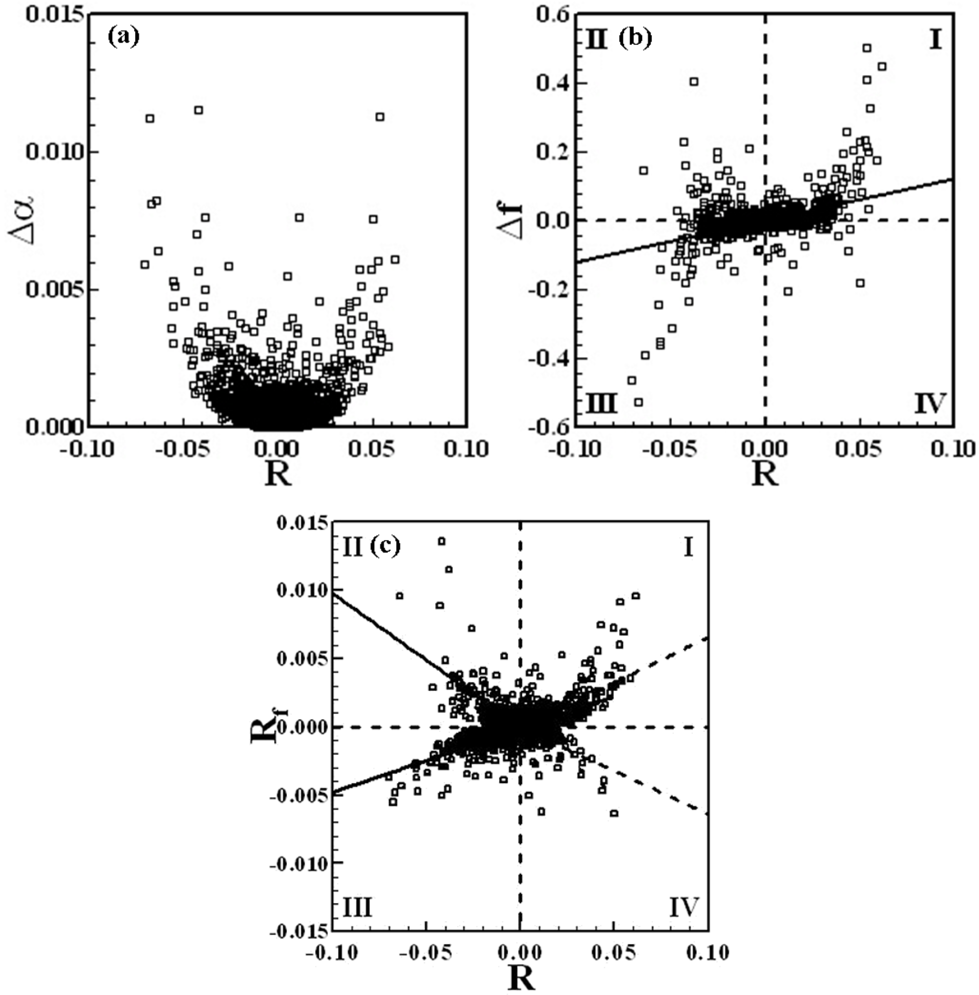

Fig. 3. (a) Point distribution of $\Delta\alpha$ vs. R (b) Point distribution of Δf vs. R. (Note the vertical and the horizontal dashed lines divide the plot into four quadrants and the solid straight line denotes the best fit of Δf as function of R) and (c) Distribution of R_f vs. R. (Note the dashed and the solid oblique lines denote best fits of R_f as functions of R in each quadrant).

tendency. Thus, investors receive vague and ambiguous signals regarding the tendency of the index. As a result, the price is equally likely to increase or decrease on the following day. In quadrant (III), *i.e.* $R < 0$, $R_f < 0$, today's closing price is lower than that of yesterday and the probability of today's price being lower than the mean price is higher than it being above. This condition implies that the price has been running at a low level for a relatively long time; thus the investors receive an obvious "over-sold" signal. As a result, they are more likely to acquire financial assets, so the price will subsequently increase. Finally, in quadrant (IV), *i.e.* $R > 0$, $R_f < 0$, the price has been running at a low level for a relatively long time, but today's closing price is higher than yesterday's. This provides an obvious signal for a potential upturn in the price movement. As a result, investors may seek to increase the level of their holdings; thus the price is likely to increase. However, this phenomenon could not be seen completely in Taiwan's stock market and we will describe this point in more detail in the final part of this section.

Figures 4(a) and 4(b) present histograms showing the average values of Δf and R_f, respectively, for different ranges of R. Figure 4(a) shows that the average value of Δf is positive when $R > 0$, but is generally negative when $R < 0$. However, an exception to this trend takes place over the interval $-0.04 < R \leq -0.03$, for which the average value of Δf is positive (see the inset in Figure 4(a)). The increasing value of Δf with increasing R shown in Figure 4(a) appears to be a market dependent phenomenon since this finding is consistent with that reported for the Hang Seng stock index in Hong Kong, but contradicts that observed for the SSEC index in Shanghai [4]. Figure 4(b) shows that the tendency of the risk indicator R_f to vary with R is broadly the same as that observed in Figure 4(a) for Δf. Wei and Huang [4] reported that the correlation between R_f and R was stronger than that between Δf and R. In other words,

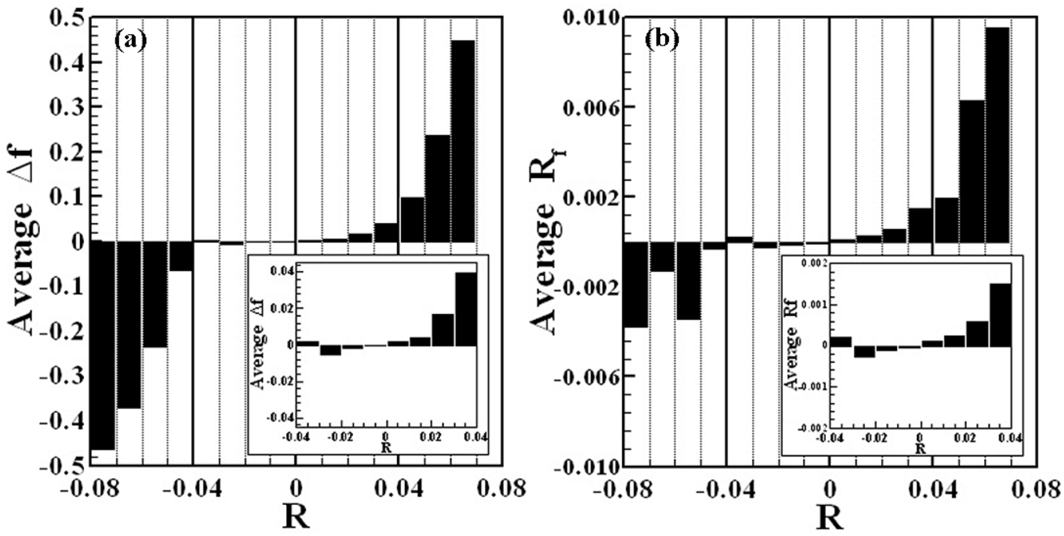

Fig. 4. (a)Variations of the average Δf with R and (b) of the average R_f with R.

when $|R|$ is large, the value of $|R_f|$ will also be large and the heights of the bars in the histogram will vary in direct proportion with changes in the value of R. However, for the Taiwanese stock market, a comparison of Figures 4(a) and 4(b) indicates that the correlation between Δf and R is stronger than that between R_f and R. Furthermore, Wei and Huang found that when $R < 0$ and $|R|$ was large, the value of $|R_f|$ was relatively larger than that of $|R_f|$ when $R > 0$ and and $|R|$ was large. In other words, it appears that for the SSEC index, R_f is more sensitive to changes in R when the price experiences a greater reduction than when the price experiences a greater increase. However, Figure 4(b) suggests that the TAIEX index exhibits the opposite trend and overall, Figure 4 shows that the variations of both Δf and R_f with R appear to be market dependent.

According to Refs. 1 and 2, the gain probability ($G\%$) and the index increase probability ($N\%$) are defined respectively as

$$G\% = \frac{R_+ \times n_- \times 100}{R_+ \times n_+ + |R_- \times n_-|}, \tag{11}$$

$$N\% = \frac{n_+ \times 100}{N}, \tag{12}$$

where R_+ represents the average value of all instances of R in the dataset whose values are greater than or equal to zero while R_- represents the average value of all instances of R in the dataset whose values are less than zero. In addition, n_+ is the number of days for which $R \geq 0$ and n_- is the number of days for which $R < 0$. Finally, N is the total number of days considered in the analysis. Figures 5(a)(a′) and (b)(b′) show the variations of the gain probability ($G\%$) and the index increase probability ($N\%$), respectively, over the ranges $\Delta f < \Delta f_c$ ($\Delta f_c < 0$) or $\Delta f > \Delta f_c$ ($\Delta f_c > 0$), where Δf_c is the threshold of Δf we choose. Note that in computing the results presented in these four figures, Δf is obtained in one of four different ways, *i.e.* (1) from the multifractal spectrum corresponding to the same day as that for which R is computed (situation 1), (2) from the multifractal spectrum corresponding to the previous day (situation 2), (3) from the multifractal spectrum corresponding to the previous two days (situation 3) and (4) from the multifractal spectrum corresponding to the previous three days (situation 4). For situation 1, it can be seen that $G\%$ and $N\%$ are lower than 50 % when $\Delta f_c < 0$ and greater than 50 % when $\Delta f_c \geq 0$. In other words, the results show that when the value of Δf for the current day is greater than zero, the gain probability for the day is greater than the loss probability. Conversely, when the value of Δf for the current day is less than zero, the loss probability for the day is greater than the gain probability. From inspection, we found that $G\%$ and $N\%$ had values of around 65 % $\sim$ 74 % when Δf had a positive value, but had values of around 10 % when Δf had a negative value. In Figure 5, the curves corresponding to situations 2, 3 and 4 indicate the dependences of $G\%$ and $N\%$ on the value of the previous day, the aggregated value of Δf over the previous two days and the aggregated value of Δf over the previous three days, respectively. It can be seen that the basic tendencies of $G\%$ and $N\%$ are similar to those discussed for situation 1. However, it is evident that the absolute values of $G\%$ and $N\%$ deviate less significantly from 50 % than in the former case. Overall, the results show that as the number of previous days considered in the computation of Δf increases, $G\%$ and $N\%$ increase toward 50 % when $\Delta f < 0$ and decrease toward 50 % when $\Delta f > 0$. In other words, the strength of the correlation between the index variation parameters ($G\%$ and $N\%$) and the multifractal spectrum parameter (Δf) weakens as the number of days considered in the multifractal computation increases. It can be seen that the values of $G\%$ and

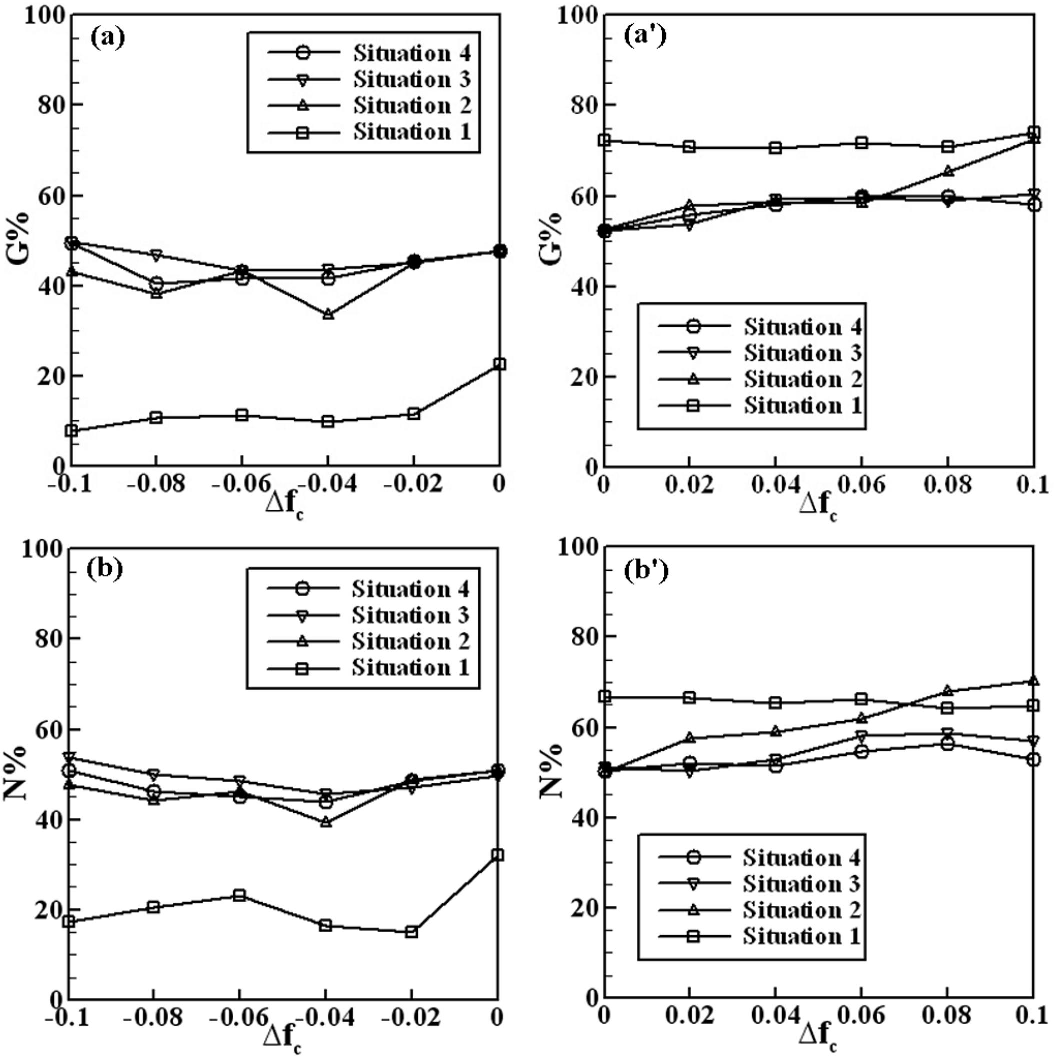

Fig. 5. (a)(a′) Variation of the gain probability $G\%$ with Δf and (b) (b′) variation of the index increase probability $N\%$ with Δf. (Note that in these figures Δf is based on the same day as the return (situation 1), the previous day (situation 2), the sum of the previous two days (situation 3) and the sum of the previous three days (situation 4).

$N\%$ are very close to 50 % when Δf is aggregated over the previous two days. Therefore, it can be inferred that the aggregated the value of Δf over the previous two days is unsuitable from a prediction perspective. The result is the same as that found for the Hang Seng stock index.

Adopting the same method as that used by Zhang [24], this study calculated the conditional probability of the index variation $\Delta I(t) = I(t+\tau) - I(t)$, where $\Delta I(t)$ is the daily index variation, $I(t)$ is the closing TAIEX value and $\tau = 1$ day. Here, the mathematical signs of $\Delta I(t)$, *i.e.* "+" or "−", represent the conditions $\Delta I > 0$ and $\Delta I < 0$, respectively. Given a sequence j composed of the signs of ΔI, the conditional probability $p(j|+)$can be defined as the probability of the predicted day having a positive index variation. Assuming that the prediction process is based on a total of M days before the predicted day, the total number of possible sign combinations is given by 2^M. For example, if M is specified as 3, eight possible ΔI sign sequences exist, namely "+++", "−++", "+−+", "−−+", "++−", "−+−", "+−−" and "−−−". Let the ratio $r(j)$ be defined as $r(j) = N_j/N$, where N_j is the number of days with a given j type condition, such as "+++" and N is the total number of days considered in the prediction process. Figure 6(a) shows the conditional probabilities and ratios associated with each of the eight possible ΔI sign sequences for the current TAIEX data. The two dashed lines in the figure indicate 50 % and 12.5 %, respectively. It is observed that the conditional probabilities $p(j|+)$ associated with the eight different sign sequences all deviate from 50 %. From inspection, the conditional probabilities of the eight sequences are found to be 54.2 %, 54.7 %, 54.4 %, 44.9 %, 48.6 %, 47.5 %, 51.8 % and 48.0 %,

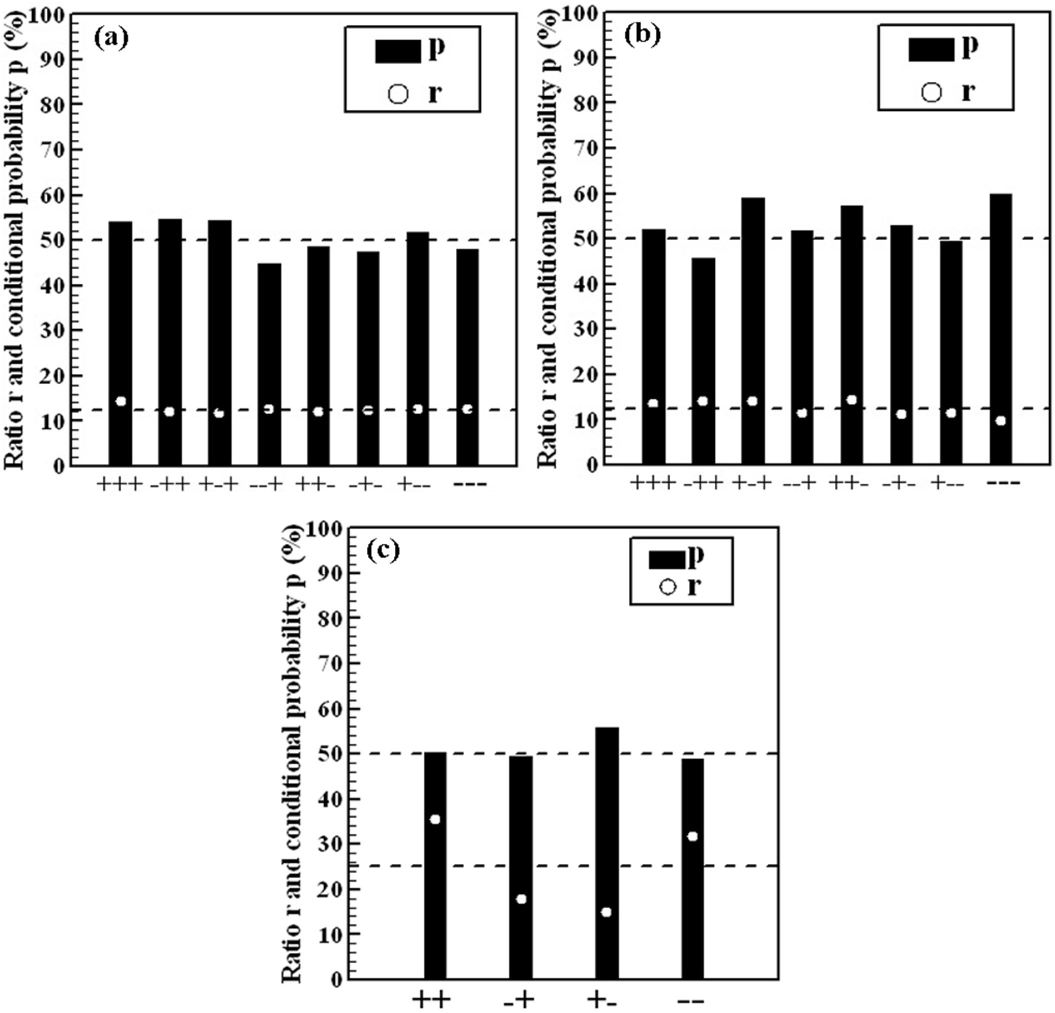

Fig. 6. Distribution of conditional probabilities (column height) and ratios (open circles) based on (a) the sign sequence of ΔI in the previous 3 days (b) the sign sequences of Δf in the previous 3 days and (c) the signs of R_f and R in the previous day.

respectively. Meanwhile, the $r(j)$ values for these eight conditions are 14.3 %, 12.0 %, 11.7 %, 12.6 %, 12.0 %, 12.3 %, 12.6 % and 12.6 %, respectively. These values of $p(j|+)$ and $r(j)$ are lower than those presented in Ref. 24 for the NYSE composite index. For the current TAIEX data, the deviations of $p_{\max}$ and $p_{\min}$ (*i.e.* the maximum and the minimum conditional probability values) from 50 % are 4.7 % and 5.1 %, respectively, while the deviations of $r_{\max}$ and $r_{\min}$ (*i.e.* the maximum and the minimum ratio values) from 12.5 % are 1.8 % and 0.8 %, respectively. In Ref. 24, the deviation of $p_{\max}$ from 50 % was higher than 10 %, while that of $p_{\min}$ from 50 % was higher than 3 %. However, the present results for $p_{\max}$ and $p_{\min}$are higher than those presented in Ref. 3 for the Hang Seng index in Hong Kong, for which it was shown that $p_{\max}$ and $p_{\min}$ deviated from 50 % by just 1.5 % and 2 %, respectively. For the Heng Seng index and the NYSE index, the minimum conditional probability, $p_{\min}$, was associated with the ΔI sign sequence "$+--$" . However, for the TAIEX data considered in the present study, $p_{\min}$ corresponds to the sign sequence "$--+$" (See Figure 6(a)). Meanwhile, $p_{\max}$ was associated with the sign sequence "$+-+$" for the NYSE index, "$-+-$" for the Hang Seng index and "$+++$" for the TAIEX index. According to Ref. 3, a conditional probability close to 50 % indicates that the stock market is more efficient. Comparing the conditional probabilities and the ratios of the Taiwanese stock market to those of the Hong Kong stock market, it seems that the former was less efficient over the period between 1999 to 2007.

Figure 6(b) shows the results obtained for the conditional probability $p(j'|+)$ and the ratio $r(j)$ when predicting the TAIEX variation based upon the mathematical sign of the multifractal parameter Δf. From inspection, the values of $p_{\max}$ and $r_{\max}$ are found to be 59.8 % and 14.3 %, respectively, which are slightly larger than or equal to the equivalent values in Figure 6(a), *i.e.* 54.7 % and 14.3 %. Meanwhile, $p_{\min}$ and $r_{\min}$ are found to be 45.6 % and 9.7 %, respectively, compared to values of 44.9 % and 11.7 %, in Figure 6(a). In other words, the value of $p_{\min}$ is similar in both prediction methods, but the value of $r_{\min}$ in the second method is lower than that in the first. Comparing the results presented in Figures 6(a) and 6(b), it is clear that a stronger correlation exists between the Δf sign sequence and the expected index movement. Therefore, one can infer that the Δf sign sequence provides a more reliable indication of the index movement than the ΔI sign sequence and is therefore more suitable for prediction purposes. This result is the same as that found for the Hang Seng stock index.

As discussed earlier in relation to Figure 3(c), Wei and Huang reported that the mathematical signs of R_f and R could be used to predict the price movements of the SSEC index [4]. The following discussions consider whether this finding is also applicable to the TAIEX index. Figure 6(c) summarizes the results obtained for the conditional probability $p(j'|+)$ and the ratio $r(j)$ for the current TAIEX data for the four possible combinations of R and R_f. Wei and Huang argued that when $R > 0$ and $R_f > 0$ (corresponding to case "++" in Figure 6(c)), the price will decrease. From Figure 6(c), it can be seen that the conditional probability associated with "++" is 50.5 %. In other words, when $R > 0$ and $R_f > 0$, the present results indicate that there is a slight probability that the TAIEX price index will increase. It seems that this result contradicts that reported for the SSEC. For the case where $R < 0$ and $R_f > 0$ (corresponding to "−+"in Figure 6(c)), Wei and Huang argued that the signal was ambiguous; thus the probability of a price increase was around 50 %. In Figure 6(c), it can be seen that the conditional probability for "−+" is 49.5 %, which is indeed close to 50 %. It thus seems that the two markets are similar. Given the conditions of $R < 0$ and $R_f < 0$ (corresponding to "−−" in Figure 6(c)), Wei and Huang stated that the price index would increase. However, Figure 6(c) shows that the conditional probability for this condition is 48.8 %, which suggests that the TAIEX will actually decrease. Finally, for $R > 0$ and $R_f < 0$ (corresponding to "+−" in Figure 6(c)), Wei and Huang again argued that the price would increase. This result is consistent with that shown in Figure 6(c), in which the conditional probability of a price increase given conditions of "+−" is found to be 55.7 %. Therefore, the results obtained for the TAIEX index are not entirely consistent with those present in the SSEC index in Shanghai.

IV. CONCLUSION

According to the efficient market hypothesis (EMH), stock markets exhibit a random walk behavior so the asset return has a normal distribution. As a result, future price movements can not be predicted based upon past price alone. However, EMH theory fails to explain many observations of empirical stock market studies. For example, Baptista and Caldas found that the evolution of the S&P 500 index return was typical of that of a low-dimensional recurrent deterministic system. The authors showed that the return evolution could be modeled with a reasonable prediction efficiency by using the Poincaré return time of chaotic logistic mapping trajectories [25]. Ivanova and Ausloos used the so-called variability diagram technique to analyze three financial data sets and showed that a reasonable predictive accuracy could be obtained over short-range forecasting intervals [26]. Mantegna and Stanley showed that the S&P 500 index was not a Gaussian process, but was actually described by a probability distribution whose central region could be modeled by using a Lévy stable process [27]. Kim and Yoon found that the probability distribution of stock market returns approached a Lorentz distribution rather than a Gaussian distribution [11].

The current study has employed a multifractal approach to analyze the minute-by-minute TAIEX data of the Taiwanese stock market over a period of eight years. The analysis has considered a total of 2162 trading days, which is significantly higher than that considered in the studies presented in the literature, *e.g.* 838 days in Refs. 1 and 2 and 586 days in Ref. 4. The results have shown that the return variation on a particular day is directly related to the absolute value of the multifractal parameter $\Delta\alpha$ on the same day. Furthermore, a positive value of Δf is indicative of an increasing return while a negative value of Δf is directly correlated with a decreasing return. In addition, it has been shown that the gain probability ($G\%$) and the index increase probability ($N\%$) have values of around 65 – 74 % when Δf has a positive value, but fall to 8 $\sim$ 32 % when Δf has a negative value. The results show that $G\%$ and $N\%$ converge toward a value of 50 % irrespective of the sign of Δf as the number of days considered in the computation of Δf increases Two methods have been proposed for predicting the future movement of the TAIEX index based upon the sign of Δf and the sign of the index variation parameter ΔI, respectively. The predictions obtained using the Δf sign sequence were shown to be more reliable than those obtained from the ΔI sign sequence. Comparing the conditional probabilities and the ratios of the Taiwanese stock market with those of the Hong Kong stock market, the Taiwanese stock market appears to be less efficient than the Hong Kong stock market over the period between 1999 and 2007. Finally, the relationship between the risk measurement parameter R_f, based upon both $\Delta\alpha$ and Δf and the price movement tendency has also been investigated. The results obtained for the TAIEX index are not entirely consistent with those presented in previous studies. Thus, we infer that the correlation between R_f and the price movement tendency is essentially stock market dependent.

In summary, the analyses presented in this study have shown that the multifractal spectra of the TAIEX return data contain a wealth of statistical information regarding the dynamic behavior of the Taiwanese stock market and can be used as the basis for rudimentary predictive tools aimed at modeling the future movements of the price index. However, the present results have also shown that many of the phenomena describing the properties of the index are stock market dependent. As a result, further research is required to develop universal rules capable of modeling the generic behavior of all international stock markets.

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