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On Uniform Almost Sure Representation of the Empirical Distribution of Φ-Mixing Random Vectors

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ABSTRACT

Let $\{\vec{X}_i: -\infty < i < \infty\}$ be a ϕ-mixing stationary sequence of random vectors with distribution function F and $\vec{\xi} = E(\vec{X}_1)$ exists. Let $F_n(\vec{x})$ be the corresponding empirical distribution function of $\vec{X}_1, \cdots, \vec{X}_n$, then we obtain an uniform almost sure representation of $|\{F_n(\vec{x}) - F(\vec{x})\} - \{F_n(\vec{\xi}) - F(\vec{\xi})\}|$ under different ϕ-mixing conditions on $\vec{X}_i$.

Key words: ϕ-mixing, stationary, almost sure representation, weak dependent.

INTRODUCTION AND PRELIMINARIES

Let $\{\vec{X}_i: -\infty < i < \infty\}$, $\vec{X}_i = (X_{i1}, \cdots, X_{iq})$ with $q \ge 1$ and distribution function $F(\vec{x})$, be a stationary ϕ-mixing sequence of random vectors defined on a probability space (Ω, A, P). That is if $E_1 \in M_{-\infty}^{k}$ and $E_2 \in M_{k+n}^{\infty}$, then for all $k(-\infty < k < \infty)$ and $n(n \ge 1)$,

$$|P(E_2|E_1) - P(E_2)| \le \phi(n), \quad \phi(n) \ge 0 \quad \cdots\cdots (1)$$

where $M_{-\infty}^{k}$ and M_{k+n}^{∞} are respectively the σ-fields generated by $\{\vec{X}_i: i \le k\}$ and $\{\vec{X}_i: i \ge k+n\}$, $1 \ge \phi(1) \ge \phi(2) \ge \cdots$, and. $\lim_{n\to\infty} \phi(n) = 0$.

Let $\vec{x} = (x_1, \cdots, x_q) \in R^q$ with $\mathrm{Cov}(\vec{X}_1)$ exists and $\vec{\xi} = (\xi_1, \cdots, \xi_q) = (EX_{11}, \cdots, EX_{1q}) = E(\vec{X}_1)$ be a point in R^q such that for the j-th variate of $\vec{X}_1$ (i.e., for X_{1j}), $P\{X_{1j} \le \xi_j\} = p_j$, $0 < p_j < 1$, $j = 1, \cdots, q$. We assume that in some neighborhood of $\vec{\xi}$, $F(\vec{x})$ is strictly monotonic in each of its q coordinates, and admits of a continuous density function $f(\vec{x})$, such that $0 < f(\vec{\xi}) < \infty$, and F has second order continuous partial derivatives at each point of the neighborhood of $\vec{\xi}$. Also, let us define

$$I_n(\tfrac{1}{2}) = \{\vec{x} : \|\vec{x} - \vec{\xi}\|_\infty \le cn^{-\frac{1}{2}} (\ln\ln n)^{\frac{1}{2}}\},$$

where c is a constant, $\|a\|_\infty = \max_{1\le j\le q} |a_j|$. For $\vec{x} \in I_n(\frac{1}{2})$, define, for $n \ge 1$, $F_n(\vec{x}) = \frac{1}{n}$\{Number of $\vec{X}_i$: $X_{ij} \le x_j$; $j = 1, \cdots, q$, for $i = 1, \cdots, n$\} is the empirical distribution function of $\vec{X}_1, \ldots, \vec{X}_n$. In

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Φ混合隨機向量的經驗累積分配函數一致性之表現

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摘 要

本篇論文旨在探討在不同的φ混合條件下，φ混合穩定隨機向量之經驗累積分配函數與分配函數間差距大小的一致性。

關鍵字：φ混合、穩定、幾乎處處表現、弱相關