嘉南藥理科技大學專題研究計畫成果報告

用加權最小平方法估計線性指數分配中的參數

計畫類別：■個別型計畫　　　　□整合型計畫

計畫編號：CNCS-91-07

執行期間：91 年 1 月 1 日至 91 年 12 月 31 日

計畫主持人：陸海林

共同主持人：

計畫參與人員：

執行單位：化妝品管理系(資訊管理系)(所)

中華民國　92　年　1　月　15　日

WEIGHTED LEAST-SQUARES ESTIMATION OF THE PARAMETERS OF THE LINEAR EXPONENTIAL DISTRIBUTION

用加權最小平方法估計線性指數分配中的參數

ABSTRACT

This investigation proposes an alternative to weighted least squares (WLS) procedure for estimating the parameters of the linear exponential distribution. The simulation results demonstrate that the novel method is superior to that of ordinary least square(OLS) method, and is not significantly different from that of maximum likelihood estimate. Furthermore, the novel method is also simple and easier to understand.

KEY WORD: Linear exponential distribution, weighted least-squares estimator, order statistics, Monte Carlo simulation.

INTRODUCTION

The linear exponential distribution with their hazard rates varying as a linear function form one such family of distribution. Some authors had ever suggested that a distribution with its hazard rate function being a lower-order polynomial is useful in the field of life-testing and reliability (Bain , 1974; Gross and Clark, 1975; and Lawless 1982). The linear exponential distribution with their hazard rates varying as a linear function form one such family of distribution. Hence, this distribution can be widely used in life-testing and reliability problems. The probability density function, cumulative distribution function, and hazard function of the linear exponential distribution are, respectively

$$f(x) = (\lambda_1 + \lambda_2 x)\exp[-(\lambda_1 x + \lambda_2 x^2)],$$

$$\mathrm{F}(x) = 1 - \exp[-(\lambda_1 x + \lambda_2 x^2)]$$

And $x>0$, $\lambda_1 > 0$, $\lambda_2 \geq 0$. \\

Bain (1974) provided a simple least squares method and maximum likelihood method to estimating the parameters the linear exponential distribution. Balakrishnan and Malik(1986) derived some recurrence relations for both single and product moments of order statistics. Ashour and Yousse(1991) investigate Bayesian estimation under type II censoring scheme. Sen and Bhattacharyya(1995) provided the EM algorithm for maximum likelihood estimate, a finite-sample exact confidence procedures and asymptotic efficiently of the least square type estimates relative to the MLE's. In this investigation, proposes a novel method of estimating the parameters of the Linear exponential distribution.

2. METHOD

2.1. Unweighted least squares type estimation

The linear exponential parameters λ_1 and λ_2 are easily obtained from least-squares analysis of the linearity form of (1) :

$$-\ln[1-F(x)]= \lambda_1 x+ \lambda_2 x^2 .$$

Bain(1974) suggested linear regression analysis as a possible method of parameters estimation. The linear regression analysis can be performed on this equation in which the F-value are assigned on basis of the position i of an observation among n ordered x-value that from a set of observations.

2.2. Maximum likelihood estimation

The maximum likelihood estimators(MLE) of the parameters is also proposed by Bain(1974).

2.3. Weighted least squares type estimation

Owing to the linear exponential distribution is an extension of the exponential distribution. Therefore, a simple transformation of a random variable with the linear exponential distribution may become standard exponential distribution. Actually, the left hand side of (2)

$$-\ln[1-F(x)]= \lambda_1 x+ \lambda_2 x^2$$

is a standard exponential distribution. Let g(Z)=Z=-ln[1-F(x)], then equation (3) can be rewritten as

$$g(Z_{(i)}) = Z_{(i)} = \lambda_1 x+ \lambda_2$$

We have $$E(Z_{(i)}) = \sum_{j=1}^{i} \frac{1}{n-j+1} ,$$

And $$V(Z_{(i)}) = \sum_{j=1}^{i} \frac{1}{(n-j+1)^2}$$

(see Balakrishnan and Cohen 1991 p. 35).

Bergman(1986) emphasized that assuming the same weigh for each datum point of the similar equation as (2) with Weibull distribution is erroneous and that a weight function should be used in performing the simple linear regression. Because the Weibull distribution has their hazard rate with linear function form, Bergman's the hypothesis should be extended to the linear exponential distribution. Therefore, a weight function should be adopted in performing the simple linear regression of the linear exponential distribution. In this investigation, we propose the weighted function

$$w_i = \frac{1}{\operatorname{var}(Z_{(i)})} = \sum_{j=1}^{i} \frac{1}{(n-j+1)^2}$$

3. A simulated study

This section uses a simulation study to evaluate the performance of the estimators using OLS, WLS(the novel method) and MLE. Appropriate criteria are essential for choosing the optimum method. The standard deviation is related to the precision of the estimators with estimators smaller standard deviations being considered better. And then, mean square error(MSE) incorporates two components, one measuring the variability of the estimator which is standard deviation(precision), and the other measuring its bias(accuracy). An estimator that has good MSE properties has small combined variance and bias. Therefore, we consider the estimators better for having smaller MSE. That is, the small the MSE is ,the high the probability of the estimators approaching the expected values of the parameters is.

4. CONCLUSION

This investigation uses a simple method to construct weight factors base on the knowledge of the characteristics of the exponential distribution and the relation with linear exponential distribution. Simulation results demonstrate that the novel method is more precise and has smaller variance than the ordinary least square method. The novel method and the maximum likelihood estimates do not differ significantly in terms of precision and bias for moderate and large sample size. For different data structure such as censoring scheme, the MLE method is intractable and need to calculated iterative by numerical method. For different models, the OLS method must select the value c to fit $F(x_{(i)})$. Meanwhile, the novel method does not need to worry about the problem of setting c to fit $F(x_{(i)})$. Therefore, the novel method is more appropriate and practical.

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