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Research

A Note on Weighted Least-squares Estimation of the Shape Parameter of the Weibull Distribution

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This study proposes an alternative to the weighted least-squares (WLS) procedure for estimating the shape parameter of the Weibull distribution. Bergman (Journal of Materials Science Letters 1986; 5:611–614), Faucher and Tyson (F&T) (Journal of Materials Science Letters 1988; 7:1199–1203) suggested using different WLS approaches for Weibull parameters. However, the simulation results show that the novel approach is better than that of Bergman, and is not significantly different from that of F&T. Furthermore, the novel approach is also simpler and easier to comprehend. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: Weibull distribution; weighted least-squares estimator; order statistics; Monte Carlo simulation; shape parameter

1. INTRODUCTION

The Weibull distribution is one of the most popular and widely used distributions in life testing and reliability studies. The distribution was proposed by Weibull[1], and its application to various failure situations was further discussed by Weibull[2]. Among other applications, this distribution has been used to investigate the fatigue life of ball bearings, describe electron tube failures and study the yield strength of Bofors steel (see Knezevic[3]). The probability density function and cumulative distribution function are, respectively,

$$\begin{aligned} f(x) &= \beta\alpha^{-\beta}x^{\beta-1}\exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \\ F(x) &= 1-\exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \end{aligned} \tag{1}$$

where $x > 0$, $\alpha > 0$, $\beta > 0$ and are referred to as the scale and shape parameters, respectively.

The Weibull distribution hazard function is monotone increasing if $\beta > 1$, decreasing if $\beta < 1$ and constant for $\beta = 1$. When $\beta = 1$, the Weibull distribution is the simple exponential distribution, once widely used as a product life distribution but later found to be inadequate for many products. Some recent works on estimating

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and testing Weibull parameters include Bergman[4]; Faucher and Tyson (F&T)[5]; Sinha and Guttman[6]; Chaudhuri and Chandra[7]; Ishioka and Nonaka[8]; Langlois[9]; Lockhart and Stephens[10]; Hossain and Howlader[11]; Drapella and Kosznik[12]; and Hung[13]. This study is motivated by Johnson *et al.*[14], who established a simple linear relation for estimating the parameters by an approximation method. Recently, a similar approximation method was introduced by Hossain and Howlader[11], who provide unweighted least-squares estimation of Weibull parameters. In fact, the two Weibull parameters α and β are easily obtained from least-squares analysis of the linearity form of Equation (1):

$$\ln\{-\ln[1 - F(x)]\} = -\beta \ln \alpha + \beta \ln x \tag{2}$$

Thus, linear regression analysis can be performed for this equation, where the F-values are assigned based on position i of an observation among n ordered x-values that form a set of observations. Supposing that X_1, $X_2, \ldots, X_n$ form a random sample from Equation (1), and that $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ are the order statistics, then Equation (2) can be rewritten as

$$y_i = \ln\{-\ln[1 - F(x_{(i)})]\} = -\beta \ln \alpha + \beta \ln x_{(i)}, \quad i = 1, \ldots, n \tag{3}$$

where $x_{(1)} < x_{(1)} < \cdots < x_{(n)}$ are observed ordered observations.

The estimator of $F(x_{(i)})$ can be considered to follow the mean rank estimator

$$\hat{F}(x_{(i)}) = \frac{i}{n+1}$$

and the median rank estimator

$$\hat{F}(x_{(i)}) = \frac{i - 0.3}{n + 0.4}$$

In reality, the two estimators can be derived from

$$\hat{F}(x_{(i)}) = \frac{i - c}{n - 2c + 1}, \quad 0 \le c \le 1$$

(see D'Agostino and Stephens[15]) by setting c as 0 and 0.3, respectively.

Bergman[3] emphasized that it is unreasonable for x_i to have the same weight in Equation (3) and proposed that a weight function should be used in performing the linear regression. The weight factor Bergman proposed is

$$W_i = [(1 - \hat{F}(x_{(i)})) \ln(1 - \hat{F}(x_{(i)}))]^2 \tag{4}$$

F&T[5] also considered Equation (3) and used a continuous curve to obtain the asymptotic weight factor, which can be expressed as follows:

$$W_i = 3.3\hat{F}(x_{(i)}) - 27.5[1 - (1 - \hat{F}(x_{(i)}))^{0.025}] \tag{5}$$

Recently, Drapella and Kosznik[12] suggested that y_i in Equation (3) can be estimated from the Weibull probability paper by using the empirical points, and α, β can be estimated with the least-squares method without including the weight factor. Drapella and Kosznik proposed

$$\hat{y}_i = \frac{n!}{(i-1)!(n-i)!} \sum_{v=0}^{i-1} (-1)^v \frac{(i-1)!}{v!(i-1-v)!} \frac{-0.5774 - \ln(n - i + v + 1)}{n - i + v + 1}$$

as the estimator of y_i and concluded that their rule estimator of β is unbiased, but has slightly greater variance than the ordinary least-squares (OLS) procedure (without weight). Hung[13] proposed a method similar to

Bergman's in Equation (6). Moreover, Hung suggested that the weight factor is given by

$$W_i = \frac{[(1-\hat{F}(x_{(i)}))\ln(1-\hat{F}(x_{(i)}))]^2}{\sum_{i=1}^{n}[(1-\hat{F}(x_{(i)}))\ln(1-\hat{F}(x_{(i)}))]^2} \tag{6}$$

Furthermore, the mean-squared error of Hung's estimator is smaller than those of Drapella and Kosznik[12] in all circumstances. In reality, the denominator of Hung's estimator is constant. Therefore, Hung's estimator is the same as Bergman's. In this article, a simulation study adopts Bergman's method in preference to Hung's. This investigation proposes a new method of estimating the shape parameter of the Weibull distribution, the details of which are presented in Section 2. Moreover, Section 3 uses a simulated study to compare the novel method with that of Bergman[4] and F&T[5]. Section 4 presents a brief discussion based on the simulation results.

2. METHOD

A simple transformation of a random variable with the Weibull distribution may become a standard exponential distribution. Herein, the left-hand side of Equation (2)

$$-\ln[1-F(x)] = \left(\frac{x}{\alpha}\right)^{\beta}$$

is a standard exponential distribution. Let $Z = -\ln[1-F(x)]$, then Equation (3) can be rewritten as

$$\ln Z_{(i)} = -\beta\ln\alpha + \beta\ln x_{(i)}$$

Assume that $X_1, X_2, \ldots, X_n$ are a random sample from the Weibull distribution and that $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ are the corresponding order statistics. Therefore, $Z_{(1)} < Z_{(2)} < \cdots < Z_{(n)}$ are the corresponding order statistics from a standard exponential distribution. Hence, the mean and variance of $Z_{(i)}$ are given by

$$E(Z_{(i)}) = \sum_{j=1}^{i}\frac{1}{n-j+1} \quad \text{and} \quad \mathrm{Var}(Z_{(i)}) = \sum_{j=1}^{i}\frac{1}{(n-j+1)^2}$$

(see Balakrishnan and Cohen[16]). Moreover, from Bickel and Doksum[17]

$$\mathrm{Var}(\ln Z_{(i)}) \simeq \frac{\mathrm{Var}(Z_{(i)})}{[E(Z_{(i)})]^2}$$

Therefore, this section uses the WLS procedure and suggests the use of the weight factor

$$W_i = [E(Z_{(i)})]^2/\mathrm{Var}(Z_{(i)}) \tag{7}$$

for estimating parameters α and β of the Weibull distribution. Moreover, the weighted sum of squares is given by

$$Q = \sum_{i=1}^{n} W_i[y_i - y_i(x_i)]^2$$

where $y_i = \ln Z_{(i)}$ and $y_i(x_i) = -\beta\ln\alpha + \beta\ln x_{(i)}$, $i = 1, \ldots, n$.

Therefore, minimizing Q can obtain the WLS estimators of the parameters α and β which are given by

$$\hat{\beta} = \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} W_i y_i v_i - \sum_{i=1}^{n} W_i y_i \sum_{i=1}^{n} W_i v_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} W_i v_i^2 - (\sum_{i=1}^{n} W_i v_i)^2} \quad \text{and} \quad \hat{\alpha} = \exp\left[\frac{\hat{\beta}\sum_{i=1}^{n} W_i v_i - \sum_{i=1}^{n} W_i y_i}{\hat{\beta}\sum_{i=1}^{n} W_i}\right]$$

where $v_i = \ln x_{(i)}$, $i = 1, \ldots, n$.

When the data are censored, the WLS procedure can also be employed to estimate α and β by using the similar technique. This technique for estimating α and β with the WLS procedure is not discussed here.

Table I. Summary of the methods investigated

Method	Equation for c	Equation for W_i
1	0	(4)
2	0.3	(4)
3	0.5	(4)
4	0	(5)
5	0.3	(5)
6	0.5	(5)
7	—	(7)

3. A SIMULATED STUDY

This section uses a simulation study to evaluate the performance of the estimators using the novel method, Bergman and F&T. The methods of Bergman[4] and F&T[5] estimate $F(x_{(i)})$ using the different values of c ($=0$, 0.3, 0.5). Table I summarizes all of the methods.

For this, we propose a Monte Carlo study of 5000 randomly generated samples, for each sample size ranging from 5 to 50. These samples were generated under the true values of $\alpha = 1$ and $\beta = 10$. Appropriate criteria are essential for choosing the best method. The standard deviation is directly related to the precision of the estimators. Therefore, we consider those estimators having smaller standard deviations to be better. For all the methods in Table I, we obtain the estimates of β, say $\hat{\beta}^{(1)}, \hat{\beta}^{(2)}, \ldots, \hat{\beta}^{(5000)}$ and calculate $E(\hat{\beta})$, $\hat{S}^2(\hat{\beta})$, and $\widehat{MSE}(\hat{\beta})$ after 5000 Monte Carlo trials, where

$$E(\hat{\beta}) = \frac{1}{5000}\sum_{i=1}^{5000}\hat{\beta}^{(i)}$$

$$\hat{S}^2(\hat{\beta}) = \frac{1}{4999}\sum_{i=1}^{5000}(\hat{\beta}^{(i)} - E(\hat{\beta}))^2$$

$$\widehat{MSE}(\hat{\beta}) = \frac{1}{5000}\sum_{i=1}^{5000}(\hat{\beta}^{(i)} - \beta)^2$$

The results for $\hat{\alpha}$ are omitted here owing to space considerations and because the accuracy of the estimators of α is significantly higher than those of β with a very small bias (<1% even in the worst-case scenario). That is, β is the key parameter for assessing these methods. Hence, the following discussion only focuses on the estimators of β. F&T[5] noted that the most accurate WLS estimate is that with the weight factor $3.3\hat{F}(x_{(i)}) - 27.5[1 - (1 - \hat{F}(x_{(i)}))^{0.025}]$ when $c = 0.3$ (method 5 in this study). According to Table II the $\widehat{MSE}(\hat{\beta})$ of methods 1, 2 and 3, which are based on the weight factor $[(1 - \hat{F}(x_{(i)}))\ln(1 - \hat{F}(x_{(i)}))]^2$, are slightly higher than that of method 7 (the novel method) when $9 \leq n$. However, no significant difference exists between the novel method and methods 1, 2 and 3 regarding $\hat{S}(\hat{\beta})$. The difference in $\widehat{MSE}(\hat{\beta})$ (or $\hat{S}(\hat{\beta})$) between the novel method and methods 4, 5 and 6, which have $3.3\hat{F}(x_{(i)}) - 27.5[1 - (1 - \hat{F}(x_{(i)}))^{0.025}]$ as the weight factor, is <3%. Hence, no significant difference in $\widehat{MSE}(\hat{\beta})$ exists between the novel method and F&T[5]. Furthermore, the difference in $\hat{S}(\hat{\beta})$ between the novel method and method 5 is nearly equal. Therefore, almost no difference exists between the novel method and method 5 provided they are applied to complete data. Simulation using censored data is also considered here. Table III lists the data scheme where a sample of size 20 contains six or ten pieces of observed data.

Under these data structures, $\hat{E}(\hat{\beta})$, $\hat{S}^2(\hat{\beta})$ and $\widehat{MSE}(\hat{\beta})$ are also calculated using the same procedure as with complete data and listed in Table IV. The results listed in Table IV differ depending on the locations of the observed data. Table IV indicates that the simulation results for censored data are in accordance with those for complete data.

Table II. Results of the estimation of β for seven methods and different sample sizes: average and standard deviation $\hat{\beta}/\beta \pm \hat{S}(\hat{\beta})/\beta$

	Method						
n	1 Bergman for $c = 0$	2 Bergman for $c = 0.3$	3 Bergman for $c = 0.5$	4 F&T for $c = 0$	5 F&T for $c = 0.3$	6 F&T for $c = 0.5$	7 Our method
5	0.869 ± 0.486 (0.536)	0.994 ± 0.559 (0.559)	1.105 ± 0.627 (0.636)	0.886 ± 0.495 (0.508)	1.022 ± 0.571 (0.572)	1.147 ± 0.643 (0.659)	0.947 ± 0.530 (0.532)
6	0.859 ± 0.418 (0.441)	0.969 ± 0.474 (0.475)	1.064 ± 0.526 (0.530)	0.873 ± 0.423 (0.442)	0.992 ± 0.480 (0.480)	1.099 ± 0.532 (0.541)	0.927 ± 0.451 (0.457)
7	0.854 ± 0.348 (0.377)	0.953 ± 0.392 (0.395)	1.036 ± 0.434 (0.436)	0.864 ± 0.348 (0.374)	0.971 ± 0.391 (0.392)	1.065 ± 0.431 (0.435)	0.913 ± 0.371 (0.381)
8	0.858 ± 0.319 (0.349)	0.949 ± 0.358 (0.361)	1.023 ± 0.394 (0.395)	0.866 ± 0.317 (0.345)	0.964 ± 0.354 (0.356)	1.049 ± 0.387 (0.390)	0.911 ± 0.335 (0.347)
9	0.859 ± 0.296 (0.328)	0.943 ± 0.330 (0.335)	1.008 ± 0.362 (0.362)	0.864 ± 0.291 (0.337)	0.954 ± 0.321 (0.325)	1.031 ± 0.349 (0.350)	0.906 ± 0.308 (0.322)
10	0.864 ± 0.272 (0.304)	0.941 ± 0.302 (0.308)	0.999 ± 0.330 (0.330)	0.866 ± 0.265 (0.297)	0.950 ± 0.291 (0.296)	1.020 ± 0.314 (0.315)	0.906 ± 0.280 (0.296)
12	0.879 ± 0.246 (0.274)	0.946 ± 0.271 (0.277)	0.996 ± 0.294 (0.294)	0.878 ± 0.236 (0.266)	0.953 ± 0.256 (0.261)	1.014 ± 0.274 (0.275)	0.913 ± 0.249 (0.263)
14	0.891 ± 0.215 (0.254)	0.951 ± 0.252 (0.256)	0.995 ± 0.271 (0.271)	0.886 ± 0.220 (0.244)	0.953 ± 0.232 (0.236)	1.008 ± 0.246 (0.246)	0.924 ± 0.228 (0.242)
16	0.900 ± 0.213 (0.236)	0.953 ± 0.231 (0.236)	0.991 ± 0.246 (0.246)	0.893 ± 0.199 (0.226)	0.955 ± 0.213 (0.218)	1.004 ± 0.225 (0.225)	0.924 ± 0.210 (0.223)
18	0.910 ± 0.206 (0.225)	0.958 ± 0.223 (0.227)	0.992 ± 0.237 (0.237)	0.900 ± 0.188 (0.213)	0.957 ± 0.201 (0.205)	1.002 ± 0.211 (0.211)	0.930 ± 0.200 (0.211)
20	0.916 ± 0.192 (0.210)	0.960 ± 0.207 (0.211)	0.990 ± 0.218 (0.218)	0.905 ± 0.175 (0.199)	0.957 ± 0.185 (0.190)	0.998 ± 0.194 (0.194)	0.933 ± 0.186 (0.197)
30	0.940 ± 0.163 (0.173)	0.971 ± 0.172 (0.174)	0.991 ± 0.178 (0.178)	0.927 ± 0.143 (0.161)	0.966 ± 0.150 (0.153)	0.996 ± 0.155 (0.155)	0.950 ± 0.151 (0.159)
40	0.955 ± 0.144 (0.151)	0.977 ± 0.149 (0.151)	0.993 ± 0.153 (0.154)	0.941 ± 0.125 (0.138)	0.973 ± 0.130 (0.133)	0.996 ± 0.133 (0.133)	0.961 ± 0.132 (0.138)
50	0.964 ± 0.129 (0.134)	0.982 ± 0.133 (0.135)	0.995 ± 0.136 (0.137)	0.951 ± 0.111 (0.121)	0.977 ± 0.114 (0.117)	0.996 ± 0.117 (0.117)	0.968 ± 0.116 (0.120)

Note: the value in parentheses is $\sqrt{\widehat{MSE}(\hat{\beta})}/\beta$.

Table III. The different cases of censored data

Case	The location of the observed data	Case	The location of the observed data
A1	2,4,6,8,10,12,14,16,18,20	B1	1,3,7,11,15,19
A2	1,3,5,7,9,11,13,15,17,19	B2	1,5,10,16,18,20
A3	1,2,3,4,5,6,7,8,9,10	B3	3,6,9,13,15,18
A4	3,4,5,6,7,8,9,10,11,12	B4	4,5,9,10,17,20
A5	5,6,7,8,9,10,11,12,13,14	B5	5,10,15,17,18,19
A6	7,8,9,10,11,12,13,14,15,16	B6	1,2,3,4,5,6
A7	9,10,11,12,13,14,15,16,17,18	B7	10,11,12,13,14,15
A8	11,12,13,14,15,16,17,18,19,20	B8	15,16,17,18,19,20
—	—	B9	8,9,10,11,12,13
—	—	B10	12,13,14,15,16,17

4. CONCLUSION

Bergman[4] and F&T[5] demonstrated that WLS is more accurate and has smaller variance than OLS without weight in parameter estimators. Therefore, there is no comparison between the novel method and the OLS

Table IV. Results of the estimation of β for seven methods and different censored data: average and standard deviation $(\hat{\beta}/\beta \pm \hat{S}(\hat{\beta})/\beta)$

Observation position	Method: 1 Bergman for $c=0$	2 Bergman for $c=0.3$	3 Bergman for $c=0.5$	4 F&T for $c=0$	5 F&T for $c=0.3$	6 F&T for $c=0.5$	7 Our method
A1	0.913 ± 0.181 (0.215)	0.963 ± 0.210 (0.214)	0.993 ± 0.222 (0.222)	0.901 ± 0.170 (0.201)	0.965 ± 0.192 (0.195)	1.008 ± 0.200 (0.200)	0.949 ± 0.191 (0.198)
A2	0.918 ± 0.199 (0.215)	0.963 ± 0.213 (0.216)	0.994 ± 0.223 (0.223)	0.909 ± 0.189 (0.211)	0.961 ± 0.200 (0.204)	0.999 ± 0.208 (0.208)	0.925 ± 0.198 (0.209)
A3	0.905 ± 0.324 (0.338)	0.969 ± 0.353 (0.355)	1.015 ± 0.378 (0.378)	0.893 ± 0.325 (0.342)	0.966 ± 0.348 (0.349)	1.023 ± 0.367 (0.367)	0.897 ± 0.326 (0.342)
A4	0.952 ± 0.338 (0.342)	0.993 ± 0.353 (0.353)	1.022 ± 0.364 (0.365)	0.956 ± 0.338 (0.341)	0.998 ± 0.351 (0.353)	1.029 ± 0.364 (0.365)	0.963 ± 0.341 (0.343)
A5	0.971 ± 0.337 (0.339)	1.004 ± 0.349 (0.349)	1.027 ± 0.357 (0.358)	0.975 ± 0.337 (0.338)	1.008 ± 0.348 (0.348)	1.031 ± 0.356 (0.358)	0.986 ± 0.341 (0.341)
A6	0.976 ± 0.327 (0.328)	1.006 ± 0.337 (0.337)	1.028 ± 0.344 (0.344)	0.979 ± 0.326 (0.326)	1.010 ± 0.336 (0.336)	1.031 ± 0.343 (0.344)	0.998 ± 0.332 (0.332)
A7	0.970 ± 0.320 (0.321)	1.003 ± 0.331 (0.331)	1.026 ± 0.339 (0.340)	0.973 ± 0.317 (0.318)	1.007 ± 0.328 (0.328)	1.032 ± 0.336 (0.338)	1.004 ± 0.328 (0.328)
A8	0.939 ± 0.296 (0.303)	0.987 ± 0.319 (0.319)	1.022 ± 0.338 (0.339)	0.929 ± 0.291 (0.299)	0.990 ± 0.309 (0.309)	1.041 ± 0.325 (0.327)	1.000 ± 0.308 (0.308)
B1	0.913 ± 0.197 (0.216)	0.964 ± 0.211 (0.214)	0.999 ± 0.222 (0.222)	0.908 ± 0.195 (0.216)	0.966 ± 0.205 (0.208)	1.009 ± 0.213 (0.213)	0.927 ± 0.203 (0.216)
B2	0.922 ± 0.191 (0.206)	0.978 ± 0.209 (0.211)	1.016 ± 0.226 (0.227)	0.896 ± 0.183 (0.210)	0.962 ± 0.195 (0.199)	1.017 ± 0.206 (0.207)	0.934 ± 0.192 (0.203)
B3	0.941 ± 0.223 (0.231)	0.978 ± 0.233 (0.234)	1.006 ± 0.240 (0.240)	0.944 ± 0.219 (0.226)	0.983 ± 0.228 (0.229)	1.011 ± 0.235 (0.235)	0.964 ± 0.225 (0.228)
B4	0.939 ± 0.208 (0.217)	0.982 ± 0.224 (0.225)	1.014 ± 0.238 (0.239)	0.932 ± 0.195 (0.207)	0.985 ± 0.206 (0.207)	1.029 ± 0.215 (0.217)	0.977 ± 0.205 (0.206)
B5	0.952 ± 0.224 (0.229)	0.990 ± 0.233 (0.234)	1.016 ± 0.240 (0.240)	0.947 ± 0.222 (0.229)	0.986 ± 0.232 (0.232)	1.014 ± 0.239 (0.239)	0.982 ± 0.229 (0.229)
B6	0.908 ± 0.502 (0.513)	1.007 ± 0.558 (0.558)	1.086 ± 0.610 (0.616)	0.913 ± 0.519 (0.526)	1.023 ± 0.575 (0.575)	1.114 ± 0.621 (0.631)	0.916 ± 0.521 (0.527)
B7	1.062 ± 0.577 (0.580)	1.093 ± 0.594 (0.601)	1.115 ± 0.606 (0.613)	1.064 ± 0.578 (0.582)	1.096 ± 0.595 (0.603)	1.118 ± 0.607 (0.619)	1.087 ± 0.590 (0.597)
B8	0.942 ± 0.448 (0.452)	1.012 ± 0.484 (0.484)	1.067 ± 0.517 (0.522)	0.953 ± 0.472 (0.474)	1.041 ± 0.514 (0.516)	1.121 ± 0.552 (0.565)	1.058 ± 0.511 (0.514)
B9	1.056 ± 0.583 (0.586)	1.087 ± 0.601 (0.607)	1.109 ± 0.613 (0.623)	1.057 ± 0.583 (0.586)	1.089 ± 0.601 (0.607)	1.111 ± 0.613 (0.623)	1.072 ± 0.592 (0.596)
B10	1.047 ± 0.565 (0.567)	1.083 ± 0.584 (0.590)	1.108 ± 0.598 (0.608)	1.051 ± 0.567 (0.570)	1.087 ± 0.587 (0.593)	1.113 ± 0.601 (0.612)	1.088 ± 0.587 (0.593)

Note: the value in parentheses is $\sqrt{\widehat{MSE}(\hat{\beta})}/\beta$.

without weight here. This investigation uses a simple method to construct weight factors based on the knowledge of the characteristics of exponential distribution and the relation with the Weibull distribution. Simulation results demonstrate that the novel method is more precision and has smaller variance than Bergman's WLS estimators. The proposed method and that of F&T[5] do not differ significantly in terms of precision. For different models, F&T[5] must find the approximate function (4) as a weight factor and must select the value c to fit $F(x_{(i)})$. Meanwhile, the novel method does not need to worry about the problem of setting c. Therefore, the novel method is appropriate and practical.

The difference between Hung's and Bergman's estimators is Equation (6) in Hung's paper has $\sum_{i=1}^{n}[(1-\hat{F}(x_{(i)}))(\ln(1-\hat{F}(x_{(i)})))]^2$ in the denominator. Both Bergman and Hung's methods have equal estimator for the shape parameter β. Therefore, it is not essential to have the denominator from the WLS point of view and Hung's method is unnecessary. Therefore, this study does not make any comparisons between the novel method and Hung's method.

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