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Approximation Theorems of the Empirical Distribution of the Sample Mean For Φ-Mixing Random Vectors

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ABSTRACT

Let $\{\vec{X}_i: -\infty < i < \infty\}$ be a ϕ-mixing stationary sequence of random vectors. Let $F_n(\vec{x})$ be the corresponding empirical distribution function of $\vec{X}_1, \cdots, \vec{X}_n$, and let $\bar{X}$, be the sample mean of $\vec{X}_1, \cdots, \vec{X}_n$. The object of the present investigation is to show that the asymptotic almost sure representation of $F_n(\bar{X})$. Two different orders of the remainder term, under different ϕ-mixing conditions, are obtained and used for proving functional central limit theorems, laws of iterated logarithm, Wiener processes embedding and invariant principles for $F_n(\bar{X})$.

Key words: weakly dependent random vectors; invariant principle; law of iterated logarithm; ϕ-mixing.

INTRODUCTION AND PRELIMINARIES

Let $\{\vec{X}_i: -\infty < i < \infty\}$, $\vec{X}_i = (X_{i1}, \cdots, X_{iq})$ with $q \geq 1$ be a stationary ϕ-mixing sequence of random vectors defined on a probability space (Ω, A, P). Thus if $M_{-\infty}^{k}$ and M_{k+n}^{∞} are respectively the σ-fields generated by $\{\vec{X}_i: i \leq k\}$ and $\{\vec{X}_i: i \geq k+n\}$ and if $E_1 \in M_{-\infty}^{k}$ and $E_2 \in M_{k+n}^{\infty}$, then for all $k\,(-\infty < k < \infty)$ and $n\,(n \geq 1)$,

$$|P(E_2|E_1) - P(E_2)| \leq \phi(n), \quad \phi(n) \geq 0 \quad \cdots\cdots (1)$$

where $1 \geq \phi(1) \geq \phi(2) \geq \cdots$, and $\lim_{n\to\infty} \phi(n) = 0$. We may remark that both m-dependent and autoregressive processes belong to the general class of strong-mixing or ϕ-mixing processes, see for example, Billingsley (1968).

Throughout this work, $\{\vec{X}_i: -\infty < i < \infty\}$ is a stationary sequence of ϕ-mixing randomvectors from a q-variate distribution $F(\vec{x})$ with Cov $(\vec{X}_1)$ exists, where $\vec{x} = (x_1, \cdots, x_q) \in R^q$. Let $\vec{\xi} = (\xi_1, \cdots, \xi_q) = (EX_{11}, \cdots, EX_{1q}) = E(\vec{X}_1)$ be a point in R^q such that for the j-th variate of $\vec{X}_1$ (i.e., for X_{1j}), $P\{X_{1j} \leq \xi_j\} = p_j$, $0 < p_j < 1$, $j = 1, \cdots, q$. We assume that in some neighborhood of $\vec{\xi}$, $F(\vec{x})$ is strictly monotonic in each of its q coordinates, and admits of a continuous density function $f(\vec{x})$, such

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Φ混合隨機向量樣本平均之近似理論

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摘　要

本篇論文旨在討論在不同的φ混合條件下，φ混合穩定隨機向量之樣本平均的近似表現，中央極限理論,對數反覆律及不變原理等性質。

關鍵字：弱相關隨機向量、不變原則、對數反覆律、φ混合。