嘉南藥理科技大學專題研究計畫成果報告

以拋物線作均勻誤差逼近之研究

計畫類別：個別型計畫

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二、中英文摘要

（一）計畫中文摘要。

關鍵詞: 片狀拋物線逼近; 曲線填充; 資料壓縮

本計畫提出一種作為曲線填充的方法. 在均勻誤差的條件下, 我們使用較少數目的拋物線段來逼近離散平面曲線.

（二）計畫英文摘要。

Keywords: piecewise parabolic approximation; digitized curve fitting; data compression

A new method for the representation of planar curves is proposed. Under the uniform error criterion, this method uses the minimal number of piecewise parabolic segments to approximate a discrete planar curve.

三、報告內容

(一) 前言。

A discrete planar curve is a point sequence on the XY-plane. In particular, it is called a digital (or digitized) curve on the grid plane where the coordinates of each point are integers. It is a well-known subject to fit a discrete planar curve with a continuous curve. There are various kinds of methods for doing this, according to the need of the applications.

Basically, a curve fitting problem concerns the efficient representation of the original planar curve for the underlined purpose. For example, B-spline curve fitting can produce smooth outlook for figure reconstruction or magnification. However, the equations make optimality difficult on error controlling. On the other hand, piecewise linear segments have their limitation to curved lines, though they can be computed easily. In general, the error estimation and the storage consideration both are important in many practical applications. Therefore, it is reasonable to reduce the number of segments for a curve fitting under the corresponding error criterion.

(二) 目的。

For this problem, we propose the following method:

(1) piecewise parabolic segments are used for the approximation;

(2) the endpoints between any two consecutive segments are constrained to lie on the original curve;

(3) the uniform error criterion means that the Euclidean distance from each of the original points to the approximate curve is within the specified error;

(4) the number of segments in the approximation is reduced as possible as it can be.

(三) 文獻探討。

We refer to some materials [1], [2], [3], [4], [6], [7], [8], [9], [10], [11], [12].

(四) 研究方法。

Let S={p_i=(x_i,y_i)| i=1,2,…,n} be a planar curve. For each point p_k in S, define a feasible set F_k={p_{k+1},p_{k+2},…,p_{k+j}}, where j is a determined integer that is still possible to approximate a segment from p_k to p_{k+j}. It is clear that j should be as small as possible.

For each point p_k, define two values v_k and t_k in the following. When program terminates, v_k denotes the minimal number of segments from p_1 to p_k and t_k denotes the path for which a segment from p_{tk} to p_k is approximated. Therefore, the optimal approximation of the curve from p_1 to p_k can be traced backward from p_k with the value t_k. Let m denote the current number of segments during the executing process. Corresponding to m, define the approximating set A={p_{i1},p_{i2},…,p_{i{\ell}}} in the meaning that for each point p_k in A, we can use the minimal number m of segments to approximate the points from p_1 to p_k within the error. We require any such point p_k to be in A.

The algorithm is described in the following.

```
begin{algorithm}
{Optimal Algorithm.}
procedure parabolic-approximation;
{ given a sequence of n points p_1, p_2,…, p_n, n>=3 }
initialize m:=0; B:={p_1}; v_k:=0, t_k:=0 for k=1,2,…,n.
while B\ne\emptyset do
begin
   A:=B; B:=\emptyset; m:=m+1;
(0)for each point p_k in A do
   begin
(1)find a feasible set F_k;
(2)for each point p_j in F_k do
      if v_j=0 then
      begin
(3)    try to approximate a segment from p_k to p_j;
         if (3) can be done
         then begin
                     v_j:=m; t_j:=k;
                     put p_j into B;
                     if j=n then return;
                  end; {if}
      end; {if}{for}
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  end; {for}
end; {while}
\end{algorithm}

If there are multiple processors available, part (0) can be put into parallel computing. When using a single processor, the points in this part can be processed with their indexes from big to small for efficiency. We suggest that they be processed from the inside to the outside ends alternatively; this may produce better segmentation of the fitting curve. For example, if A= {2,3,4,5,7,9} for m=1 in the first approximation, then these points are processed in the following order at the next iteration: 5, 4, 7, 3, 9, 2.

Part (1) can be obtained by the method used in ref. [8] or ref. [9]. As to part (3), we first decide a tangent direction for each point indexed from 2 to n-1 in the beginning. On testing if there is a parabola segment with endpoints p_k and p_j, we restrict the search among those possible segments which also pass one point p_h between p_k and p_j and with the prescribed tangent direction. Note that there is exactly one such possible segment for each p_h. Suppose after coordinate transformation we have p_h=(0,0) with X-axis as the tangent line, p_k=(x1,y1) with x1<0 and y1>0, p_j=(x2,y2) with x2>0 and y2>0, then the parabola passing thru p_j, p_h, p_k with X-axis as the tangent line at p_h has the following form: y=A(x+By)^2. Let C=y1/y2. Then we have two solutions for B.

B1=(x1-sqrt{C}x2)/(sqrt{C}y2-y1)

B2= -(sqrt{C}x2+x1)/ (sqrt{C}y2+y1)

However, only one solution of B corresponds to a parabola that makes p_j and p_k each separately locates on one of the two parts separated from p_h of the parabola. It is the one which satisfies (x1+By1)(x2+By2)<0. We further need to test if each point between p_k and p_j is within the specified error to this parabolic segment. Therefore, unless line segment from p_k to p_j need to be used, this way of fitting a parabola segment always works if there is just one point between p_k and p_j.

Besides this algorithm introduced here, the suboptimal approach (i.e., the scan-along method) can be considered for time consuming. That is, we start from p_1, next find the longest segment and continue the same process at the other terminal of the current segment. Though this method may not reach the minimal number of segments, it saves much time.

(五) 結果與討論。

This new method of fitting parabolic segments to a discrete planar curve is expected to perform well for curved outline. It is not supposed to be used for curves with many corners or sharp angles, to which the simple uniform linear approximation should be used.

四、參考文獻

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五、成果自評

這是一種有效的曲線填充方法. 特別針對非線性曲線可達到良好的結果.

六、可供推廣之研發成果

可提供離散資料曲線的連續表式. 以進行資料分析, 圖形輪廓之放大及資料壓縮之應用.